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MODELLING THE DEPENDENCY STRUCTURE AMONG THE BRICS MARKET RETURNS USING COPULA ARMA-EGARCH APPROACH

Abstract:

This study investigates the dependency structure among BRICS market returns using a Copula-based ARMA-EGARCH modelling framework. The stationarity tests using ADF and PP confirmed that the closing prices are non-stationary at level but stationary after first differencing. Pearson correlation and scatter plots indicate strong positive linear relationships among the markets, suggesting co-movement. Given the evidence of tail dependence, copula models are employed to capture both upper and lower tail dependencies. ARMA models were selected based on AIC, and diagnostic tests confirmed their adequacy. The hybrid ARMA-EGARCH(1,1) model was used to model volatility, capturing both positive and negative leverage effects across different markets. Residuals from these models were transformed into uniform distributions and used to fit four copulas, namely, Student-t, Frank, Clayton, and Gumbel. Results showed that the Student-t copula best captures the dependence structure, with significant dependence parameters for all pairs. Lower and upper tail dependencies were captured by Clayton and Gumbel copulas respectively, indicating asymmetric behaviour during market extremes. The findings suggest strong financial integration among BRICS markets, limiting diversification benefits and increasing contagion risk. The study recommends the use of flexible distributions and more advanced copula models like Joe-Clayton in future research to enhance Value-at-Risk estimation and risk management practices.

Keywords:

BRICS market returns, Copula, Dependency structure, Hybrid models, Volatility

JEL Classification: C10, C52, F30

1. Introduction

Volatility and dependence structure have long been pivotal concepts in financial economics, owing to their profound implications for international portfolio diversification and market integration (Engle, 2002). Volatility, characterised by Zivot and Wang (2006) as the erratic and explosive behaviour of financial markets, serves as a fundamental measure of risk. In parallel, the dependence structure defined by Ijumba (2013) as the behavioural dynamics of one market or variable in response to another, illuminates the transmission channels of financial shocks. The relationship between volatility and market dependence is particularly pronounced in emerging economies, where financial markets often exhibit asymmetric reactions to global shocks (Bekaert and Harvey, 2003). Traditional financial models, such as the capital asset pricing model (CAPM) and linear correlation measures, frequently prove inadequate in capturing nonlinear dependencies and tail-risk events (Patton, 2006), necessitating more advanced econometric methodologies. Recent innovations, including copula theory, GARCH models (Sklar, 1959; Bollerslev, 1986), and regime-switching frameworks (Hamilton, 1989), have significantly enhanced the empirical analysis of these interconnected financial phenomena.

The post-pandemic financial era has witnessed a resurgence of academic interest in market volatility and cross-asset dependence structures, particularly concerning their role in global portfolio allocation and systemic risk contagion (Bouri *et al*., 2021; Corbet *et al*., 2019). Modern financial theory conceptualises volatility not simply as a conventional risk measure, but rather as a fundamental mechanism for shock transmission, with leading scholars likening it to “the DNA of market stress” (Antonakakis and Filis, 2013). Simultaneously, the analysis of dependence structures has transcended traditional linear correlation frameworks, with current research prioritising the examination of extreme tail dependence and asymmetric spillover effects during periods of financial crisis periods (Reboredo *et al*., 2024; Mensi *et al*., 2024).

This study carries particular significance for BRICS financial markets, given the ongoing developmental trajectory of certain member economies such as South Africa. As a prominent emerging market, South Africa’s extensive network of international economic relationships renders its cross-market interactions particularly consequential for global financial dynamics. Mensah and Alagidede (2017) demonstrated that substantial trade volumes (both import and export) between BRICS nations reflect robust economic interconnectedness. To gain deeper insights into the impact of these connections, it is essential to analyse the volatility within each BRICS market individually, as well as examine the interdependencies that exist among them. Studying the volatility dynamics and dependence structure is essential for risk management, asset allocation, and portfolio diversification.

Volatility and asymmetries on individual BRICS markets are captured by using the Hybrid Autoregressive Integrated Moving Average (ARMA)-Exponential Generalised Autoregressive Conditional Heteroskedasticity (EGARCH) model, and dependence structure between the BRICS markets is/are captured by using the copula model. Having better knowledge of volatility clusters, assymtries structures and dependence structure provides policy makers with information on how to deal with future risks associated with those particular financial markets.

The Hybrid ARMA-EGARCH model’s contributions lie in its potential to handle both the hidden structures, where the joint distributions of paired datasets is modelled independently using the marginal distribution of each dataset. Furthermore the model is able to handle volatility clusters with leverage effect as well. The current study contributes to the field of econometrics, statistics and many more. The presence of a Sign Bias test makes an important contribution by pinpointing asymmetries (negative and positive leverage effect). Lastly, all that has been mentioned above is important to investors, risk managers, and policy makers.

The rest of the study is organised as follows: Section 2 presents the related literature review, Section 3 discusses the research methodology, Section 4 presents the discussion of findings and lastly Section 5 discusses the conclusion.

2. Literature review

Mustapa and Ismail (2019) performed an empirical investigation of hybrid ARIMA-GARCH modelling using S&P500 index data. Their research employed a 16-year monthly time series dataset (January 2001-January 2017) to evaluate model performance, with particular emphasis on short-term forecasting applications for investment decision-making. The study methodology involved: identifying optimal lag structures through ACF and PACF analysis, conducting parameter estimation, and comprehensive model evaluation. Their findings demonstrated that the hybrid ARIMA (2,1,2)-GARCH (1,1) specification yielded superior forecasting performance among the tested series. This optimal model specification provides valuable insights for portfolio managers seeking to enhance short-term market predictions.

Senneset and Gultvedt (2020) conducted an empirical investigation into the enhancement of stock pair correlation estimation through contemporary forecasting techniques. Their research developed a hybrid modelling framework integrating both ARIMA and GARCH components, designed to simultaneously capture linear correlation patterns and volatility clustering phenomena. Utilising time series data from the Oslo Stock Exchange spanning 2006 to Q3 2020, their comparative analysis demonstrated superior predictive performance of modern forecasting approaches relative to conventional methods. Specifically, the integrated hybrid model achieved an RMSE of 0.186 across all test periods, representing a significant improvement over the benchmark average RMSE of 0.237. The consequences of these findings, however, are equivocal, as the improvement in prediction accuracy cannot be claimed to conclusively balance the rise in implementation cost.

Vo and Ślepaczuk (2022) conducted a comparative analysis of forecasting performance between linear ARIMA models and hybrid ARIMA-GARCH specifications (specifically ARIMA-SGARCH and ARIMA-EGARCH) for S&P500 log returns. Using daily data from Yahoo Finance spanning January 1, 2000 to December 31, 2019, the study employed multiple forecast accuracy metrics including MAE, MAPE, and RMSE to evaluate model performance. The empirical results demonstrated two key findings: both hybrid models significantly outperformed the standalone ARIMA specification in predictive accuracy, but contrary to expectations, the ARIMA-EGARCH variant failed to surpass the ARIMA-SGARCH model's performance. These findings suggest that while GARCH augmentation improves forecasting capability, the choice of conditional variance specification critically impacts model effectiveness.

Mensah and Alagidede (2017) examined the dependence structure between four major African stock indices (South Africa's JSEOVER, Nigeria's NIGALSH, Egypt's EGHFINC, and Kenya's NSE20) and two developed-market benchmarks (S&P 500 and FTSE 100) using daily data from 2000 to 2014. Their findings revealed a weak financial linkage among African markets, except for South Africa; asymmetric dependence, with differing co-movements in bullish and bearish conditions; and limited spillover effects from advanced-market downturns to African equities. These results suggest that African markets (excluding South Africa) are largely insulated from developed-market financial shocks, highlighting potential diversification benefits for international investors.

Charfeddine and Benlagha (2016) conducted an empirical analysis of the time-varying dependence between major stock indices (S&P500, CAC, DAX, and FTSE 100) and twelve commodity markets. Employing daily price data spanning from July 7, 1992 to February 17, 2015, the researchers employed a rolling-window copula approach to examine how dependence structures evolved across different market volatility regimes. Their methodological framework specifically assessed dependence dynamics in low, medium, and high volatility periods. The study's key finding revealed that the Student-t copula specification demonstrated superior performance in capturing the complex dependence patterns between equity and commodity markets compared to alternative models.

3. Methodology

The study analysed weekly closing prices from the BRICS markets, including Brazil (BVSP), Russia (IMOEX), India (NSEI), China (SSE), and South Africa (JSE). The dataset covers a nine-year period from January 1, 2014, to December 31, 2023, comprising 522 observations for closing prices and 521 for returns. Data was obtained from the IRESS platform (Trading and Market Data | Software for Traders and Investors | Financial Software by IRESS). This dataset serves as the foundation for assessing the statistical tests and models employed in the study. Additionally, the analysis was extended beyond the BRICS trade bloc to meet the requirements of certain models that necessitate over 100 observations.

3.1 Preliminary analysis

Chatfield (2013) described time series analysis as the study of data points recorded at consistent, sequential time intervals. It is employed to identify patterns, model the behaviour of observed variables, and predict future values. This approach is widely used in fields such as statistics, economics, and engineering. In statistical research, conducting a preliminary analysis before empirical investigation is a common practice, as it helps quantify key characteristics of a random variable. In this study, the preliminary analysis includes descriptive statistics, stationarity and correlation testing. Pantelis and Zehtabchi (2008) defined stationarity as a property in which a process maintains a constant mean and variance over time, indicating that these statistical properties do not fluctuate. This specific form of stationarity is known as weak or second-order stationarity. For a time series to be considered weakly stationary, the following conditions must be met:

$$E(X_t) = \mu \qquad t = ,2, \ldots, \infty \tag{1}$$

$$Var(X_t) = \sigma^2 \qquad < \infty \tag{2}$$

$$Cov(X_t, X_{t-k}) = \gamma_k \qquad \text{For all } t \text{ and any } k \tag{3}$$

The stationarity of a time series is formally assessed using the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test. The ADF test, developed by Dickey and Fuller (1979), is an extension of the original Dickey-Fuller (DF) test, designed to accommodate more complex time series structures. While the DF test is applicable only to autoregressive processes of order one (AR(1)), the ADF test is preferred for cases involving higher-order correlations, as highlighted by Pantelis and Zehtabchi (2008). The following equation represents the ADF model used to evaluate the presence of a unit root:

$$\Delta r_t = \beta_1 + \beta_2 t + \delta X_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta X_{t-1} + \varepsilon_t \qquad (4)$$

where Δ is the difference operator, β_1 is a constant, β_2 is the coefficient on the time trend, and δ is the coefficient for the lagged X_{t-1}. The differences, $\Delta X_{t-1} = (X_{t-1} - X_{t-2}), \Delta X_{t-2} = (X_{t-2} - X_{t-3})$ and so on, are calculated for the time series. The selection of lagged difference terms in stationarity tests is typically determined empirically. Including an adequate number of lag terms helps eliminate serial correlation in the error term, ensuring unbiased estimates of δ. The Phillips-Perron (PP) test, introduced by Phillips and Perron in 1987, serves as an alternative to the ADF test, particularly when the latter fails to adequately address serial correlation. As noted by Brooks and Burke (2003), the PP test is especially useful for correcting serial correlation issues that may arise in the data.

Examining and comprehending the relationships among BRICS markets is crucial. While traditional correlation methods help describe these connections on a basic level, copula models provide deeper insights. Various correlation tests and graphical techniques are available to analyse associations between variables. Common tests include the Pearson correlation coefficient, Spearman rank correlation, and Kendall rank correlation, while visual tools such as scatter plots, pair plots, correlation matrix heatmaps, and correlograms can further support the analysis. However, these methods typically assume normality and linearity hat is ignorant of tail behaviour. In this study, the Pearson correlation coefficient created the Pearson correlation coefficient test after Francis Galton proposed a related concept in the 1880s will serve as the primary test, supplemented by the scatter plots for validation. The Pearson's correlation coefficient is computed using:

$$r = \frac{\sum_{i=1}^{n}(r_i - \bar{r})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(r_i - \bar{r})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \qquad (5)$$

where r, represent correlation coefficient, r_i is the first variables, y_i is the second variable, $\bar{r}$ and $\bar{y}$, is the average of the first variable and the second variable. Lastly, the scatter plot is a probability plot that is responsible for comparing statistical orders. The statistical orders compared, are theoretical (X-axis) to empirical (Y-axis). When following conditional distributions of random variables that assume normality, then the scatter plots are connected in a linear manner. The scatter plots considers an estimate which exists from the inverse of the CDF such that $\hat{F}_{P_{i,n}}^{-1}$ verses $r_i, n; i = 1,2, \dots, n$. Suppose the model is normally distributed, then the observed points are very close to a 45-degree line.

3.2 Base models

After ensuring stationarity, it is essential to check for serial correlation (non-constant conditional mean) in the returns. This helps determine the appropriate lag orders (p, q) for AR, MA, or ARMA models using ACF and PACF plots. In this study, ARMA models up to lag 5 will be considered, with model selection based on the Akaike Information Criterion (AIC). For detecting serial correlation, the Box-Ljung test (formal method) will be used, while LM-ARCH (Lagrange Multiplier) test will assess ARCH effects (non-constant conditional variance) in squared returns. Unlike ARMA models, EGARCH(1,1) will be directly applied without lag order selection, as Engle *et al.* (1990) suggest it effectively captures volatility clustering. Additionally, ACF and PACF plots of squared returns will visually confirm significant lags, supporting the presence of time-varying variance. Both formal tests and graphical methods should yield consistent conclusions.

The AR (p) captures the linear relationship between a current value and its past lagged values in a time series. It expresses the present value as a weighted sum of previous observations, where the weights represent the strength of dependence on past values. The parameter "p" determines how many prior lags are included in the model (Tsay, 2010). The general form of the AR (p) model is:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \cdots + \alpha_p r_{t-p} + \varepsilon_t \quad (6)$$

where r_t represent present time series process, α_0 is the constant intercept, $\alpha_1, \ldots, \alpha_p$ represent the coefficients associated with the specified order p, and lastly ε_t represents the error term at time t. The MA (q) captures short-term dependence in a time series by modelling the current value as a linear combination of past error terms (residuals). Unlike the AR(p) model, which relies on past observed values, the MA (q) model focuses on the impact of previous shocks or unexpected fluctuations (Tsay, 2010). The MA (q) model is given by:

$$r_t = \beta_0 + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \cdots + \beta_q \varepsilon_{t-q},\ q > 0 \quad (7)$$

where β_0 is the mean of the series and $\beta_1, \ldots, \beta_q$ represent the coefficients associated with the lagged residual errors. Since ARMA comes from AR (p) and MA (q) model, then the same properties associated with the AR (p) and MA (q) model as individual models, are the same properties used to elaborate an ARMA model. An ARMA model is given by:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \cdots + \alpha_p r_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \cdots + \beta_q \varepsilon_{t-q} \quad (8)$$

where $\alpha_1, \ldots, \alpha_p$ are AR coefficients responsible for representing the dependence on lagged values and $\beta_1, \ldots, \beta_q$ are the MA coefficients responsible for representing the dependence on the lagged residual errors. When modelling data using an ARMA process, an iterative estimation procedure is required. The methodology builds upon the same fundamental principles used for estimating standalone AR and MA models. A key step involves computing the residual sum of squares (RSS), where different parameter combinations across a defined grid are evaluated to identify those that minimise the squared error terms.

The EGARCH model was first discovered by Bollerslev in 1986 to deal with leverage effect, which can be included into the GARCH model in different ways. Nelson (1991) examined the weighted innovation given by the following equation:

$$(\eta_t) = \theta \eta_t + \gamma(|\eta_t| - E[|\eta_t|]) \quad (9)$$

where θ and γ are real constant. Thus η_t and $|\eta_t| - E[|\eta_t|]$ are zero mean is iid sequence with continuous distribution. Then continue to have $E[g(\eta_t)] = 0$ hence the asymmetry of $g(\eta_t)$ can be shown as:

$$g(\eta_t) = \begin{cases} (\theta+\gamma)\eta_t - \gamma E(|\eta_t|), & if, \quad \eta_t \geq 0 \\ (\theta-\gamma)\eta_t - \gamma E(|\eta_t|), & if, \quad \eta_t < 0 \end{cases} \tag{10}$$

It is said that the procedure is EGARCH if equation (9) is satisfied where:
$\varepsilon_t = \sigma_t \eta_t$

$$ln(\sigma_t^2) = \alpha_0 + \frac{1+\beta_1 B+\cdots+\beta_{q-1}B^{q-1}}{1-\alpha_0 B-\cdots-\alpha_p B^p} g(\eta_{t-1}) \tag{11}$$

where α_0 represents a constant and B is the back-shift operator hence $Bg(\eta_t) = g(\eta_{t-1})$. The numerator $1 + \beta_1 B + \cdots + \beta_{q-1}B^{q-1}$along with the denominator $1 - \alpha_1 B - \cdots - \alpha_p B^p$ are polynomial with zeros outside the unit cycle (Tsay, 2005). The EGARCH model can alternatively be given as:

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{|\varepsilon_{t-i}|+\gamma_i\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \, ln(\sigma_{t-j}^2) \tag{12}$$

When EGARCH model is written in the form of equation (10), then a positive ε_{t-i} which contributes $\alpha_i(1+\gamma_i)|\eta_{t-i}|$ to log volatility and a negative ε_{t-i} which contributes to $\alpha_i(1-\gamma_i)|\eta_{t-i}|$ where $\eta_{t-i} = \varepsilon_{t-i}/\sigma_{t-i}$. Therefore, the parameter γ_i is regarded as leverage effect of ε_{t-i} which comes into existence when the returns become more volatile, which happens when the price decreases (Tsay, 2005). The ARMA and EGARCH models are estimated using maximum likelihood (ML) method.

3.3 Hybrid ARMA (p, q)-EGARCH (1,1) models

Neither standalone ARMA nor EGARCH models can adequately capture all characteristics of a Gaussian white noise process. To address this limitation, this study employs a hybrid ARMA-EGARCH model rather than using these models independently. This combined approach necessitates adopting a modified z-distribution assumption for the residuals. As demonstrated by Otieno (2017). The ARMA model solely predicts the conditional mean, while the EGARCH model exclusively forecasts conditional volatility based on the leverage effects detected. Their integration enables volatility-adjusted return forecasting. The combined model effectively transforms the process into Gaussian white noise, with parameter estimation following similar procedures to the standard volatility model estimation process. A Hybrid ARMA-GARCH model follows:
$X_t \sim D(r_t; \sigma_t^2)$

where σ_t^2 is iid, D is assumed to be some density while the lag orders may vary (allowing for a nonzero unconditional mean), the conditional mean process due to ARMA has the same form as the conditional variance process due to EGARCH. The hybrid modelling approach follows a two-stage estimation procedure. In the initial phase, linear components of the time series are captured through ARMA specification. The subsequent phase employs EGARCH modelling to address volatility and leverage effect patterns in the dataset. This integrated framework enables comprehensive series analysis and enhanced predictive capability (Tan et al., 2010). In this procedure, it is anticipated that the The ARMA-EGARCHmodel's error term will be estimated under SSTD and GED,as the distribution has heavy tails, a sharp tall peak and higher kurtosis.

3.4 The Sign Bias test

The test was introduced by Engle and NG in the year 1993. The test was introduced mainly because of its ability to capture leverage effect in its complex form. The two authors define the Sign Bias test as a tool that is used to determine whether there is a systematic tendency for volatility to be higher following negative price changes as opposed to positive price changes. (Bollerslev and Zhou, 2006). Below is the Sign Bias test equation:

$$\tilde{\alpha}_t^2 = c_0 + c_1 1_{\hat{\alpha}_{t-1<0}} + c_1 1_{\hat{\alpha}_{t-1<0}} \hat{\alpha}_{t-1} + +u_t. \quad (13)$$

where $\tilde{\alpha}_t^2$ is the squared standardised residuals at time t, $\hat{\alpha}_{t-1}$ is the residuals at time $t-1$, $\hat{\alpha}_{t-1} < 0$ and $\hat{\alpha}_{t-1} \geq 0$, are indicator function that is/are equal to 1, c_0 is intercept parameter and c_1 represent leverage effect.

3.5 Copula models

Copula functions, first introduced by Sklar (1959), serve as a powerful statistical tool for modelling dependence structures between random variables. The term "copula" originates from the Latin word meaning "link" or "tie," reflecting its function of connecting marginal distributions to form a joint distribution (Kemda, 2015). According to Sklar's Theorem, a given multivariate distribution function and its univariate marginals are connected by a copula function. Regarding the bivariate distribution, univariate marginal distribution functions $F_{MN}(m,n)$ and joint distribution $F_M(m)$ and $F_N(n)$ a copula C exists (Sklar, 1959) as computed by:

$$F_{MN}(m,n) = C\big(F_M(m), F_N(n)\big) = C(r,y)$$

$$C: [0,1]^2 \to [0,1] \quad (14)$$

If C is a copula which is responsible for capturing dependence structure, and F_M and F_N are distribution functions, F_{MN} can be defined as a joint distribution with F_M and F_N. The Probability Density Function (PDF) of the bivariate distribution $F_{M,N}(m,n)$ is represented by the following equation:

$$f_{M,N}(m,n) = C\big(F_M(m), F_N(n)\big). f_M(m). f_N(n) \quad (15)$$

where the univariate marginal PDFs of M and N are $F_M(m)$ and $F_N(n)$ respectively. A copula function is necessary when the marginals are continuous, and function is special. Genest and Rivest (1993) and Jaworski (2010) both list some properties of copula. A copula is a function C from $[0.1]^2$ to $[0,1]$ in the bivariate case. The copula models is able to model, non linear , non symmetric and tail dependence structure that traditional methods like Pearsons correlation coefficient and scatter plots may unable to handle. There are two types of copula models, namely, empirical and theoretical copula. The empirical copula, which is defined in rank space, is an estimator for that distribution while the theoretical copula distribution is unknown. Since the true underlying copula distribution is typically unknown in empirical applications, this study concentrates on theoretical copula families. Specifically, the investigation will examine two principal classes of theoretical copulas, namely, Elliptical Copulas (including Gaussian and Student-t copulas) and Archimedean Copulas (such as Clayton, Gumbel, and Frank copulas). However, the Gaussian normal copula is excluded from the study. The Gaussian normal copula is unable to provide a reliable tail dependence structure as indicated by Jondeau and Rockinger (2006). These parametric copula families provide mathematically tractable frameworks for modelling dependence structures while making specific assumptions about the nature of variable relationships.

The ML estimation is the most used estimation technique for copula parameters. Therefore, in this study, the copula parameters are determined through ML estimation, namely, Canonical Maximum Likelihood (CML). The first stage is marginal distribution estimate. In order to estimate the copula parameter, a pseudo-sample of the modified observation is used. The CML uses univariate marginals that are empirical distribution functions (Giacomini, 2005). Assumed to be an *iid* random variable are the sample dataset$(m_1, m_2, m_3, \dots, m_n)$. This information is changed into uniform variates $(r_1, r_2, r_3, \dots, r_n)$. A non-parametric estimate of the empirical CDF of the marginal distribution serves as the transformation, given by:

$$\hat{r_i} = \widehat{F_I}(m_i) \tag{16}$$

3.6 Model selection

Information criteria are used when choosing the appropriate models. In this study, AIC will be used to choose the best model, However, the Log likelihood (LL) will be included as well in choosing the best copula model. The AIC is given as:

$$AIC = 2\,log(L) + 2k \tag{17}$$

where k represents the number of parameters found in the model, L represents the maximise value of the log-likelihood of the model. The model with the lowest value of AIC is then chosen to be the best and appropriate model to be used for further analysis. Note that, the model selection method is for all the proposed models of the current study.

4. Discussion of findings

This section presents and analyses the study's findings, supported by tables and figures for clarity and illustration.

4.1 Preliminary analysis

Descriptive statistics are used to summarise the key features and characteristics of the dataset. These measures include the minimum, maximum, mean, median, and standard deviation (Std. Dev.). A summary of these descriptive statistics is provided in Table 1 and Table 2.

Table 1: Results of the descriptive Statistics for the BRICS closing prices

Measure	BVSP	IMOEX	NSEI	SSE	JSE
Obs	522	522	522	522	522
Minimum	345126.8592	1204.1245	5888.2125	2145.2150	39215.1248
Maximum	128722.13654	3941.1248	17624.0011	4215.2399	74253.2290
Mean	79145.0101	2255.2513	10011.0011	2900.2122	49586.2153
Median	76551.2110	2001.2000	11561.0200	3030.0001	51200.0088
Std. Dev.	23315.0199	985.7888	3211.2145	780.2215	7452.0033

Table 2: Results of the descriptive statistics for the BRICS returns

Measure	BVSP	IMOEX	NSEI	SSE	JSE
Obs	521	521	521	521	521
Minimum	-0.2221	-0.2121	0.-1242	-0.2133	-0.1221
Maximum	0.1325	0.1129	0.9990	0.1258	0.0875
Mean	0.0021	0.0007	0.0019	0.0005	0.0025
Median	0.0012	0.0010	0.0036	0.0028	0.0023
Std. Dev.	0.0380	0.03030	0.0315	0.0200	0.0218

Table 1 and Table 2 present the descriptive statistics for the BRICS closing prices and returns. The results indicate that both the mean and the median are positive, suggesting a slight increase in the closing prices and returns over time. Additionally, the significant gap between the minimum and maximum values reflects high market variation. Understanding the concept of stationarity is crucial, as it refers to maintaining constant statistical properties over time. Conducting a stationarity test simplifies model estimation and forecasting. The ADF and PP tests were utilised, and the results of these tests are presented in Tables 3 and Table 4.

Table 3: ADF and PP test results of the BRICS closing prices

Returns	ADF	Prob.	PP	Prob.
BVSP	-2.0322	0.2731	-1.9245	0.3211
IMOEX	4.2829	1.0000	-3.4319	1.0000
NSEI	0.4922	0.9865	-3.4319	0.9922
SSE	-0.8712	0.7978	-3.4319	0.8829
JSE	-1.1239	0.7086	-3.4319	0.8474

Note: 5% significant level is used.

Table 4: ADF and PP test results of the BRICS returns

Returns	ADF	Prob.	PP	Prob.
BVSP	-34.7475	<0.0000	-67.0220	<0.0000
IMOEX	-40.9647	<0.0000	-34.8161	<0.0000
NSEI	-34.8161	<0.0000	-62.9122	<0.0000
SSE	-67.1647	<0.0000	-62.6223	<0.0000
JSE	-61.7098	<0.0000	-64.71547	<0.0000

Note: 5% significant level is used.

Tables 3 and 4 display the stationarity test results for the BRICS closing prices and returns. According to Table 3, the p-values for both the ADF and PP tests on BRICS closing prices exceed the 5% significance level. Consequently, the null hypothesis (H_0) cannot be rejected, indicating that the time series is non-stationary and requires differencing. In contrast, Table 4 reveals that the p-values for BRICS returns are below the 5% significance level, leading to the rejection of H_0. Thus, the BRICS returns are stationary. The time series plots are presented in Figure 1 and Figure 2.

Figure 1: Time series plots of the BRICS closing prices

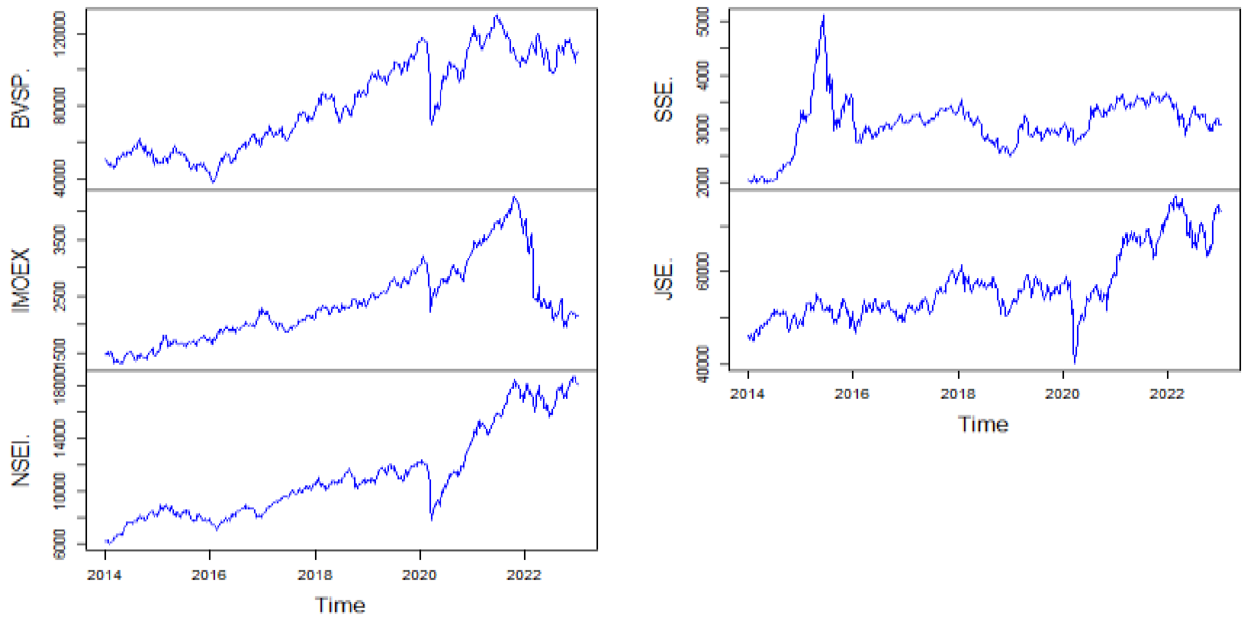


Figure 2: Time series plots of the BRICS returns

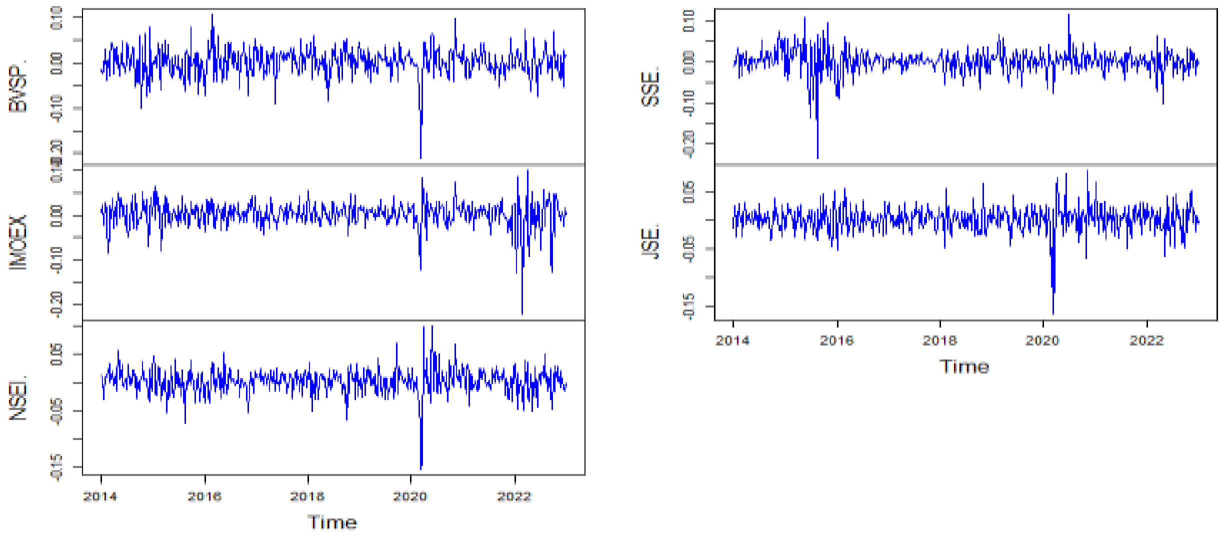


Figure 1 illustrates that all plots exhibit seasonal upward and downward movements. In contrast, Figure 2 indicates that the time series process is stationary, as the data fluctuates around zero. Additionally, the figure highlights the presence of volatility clustering, where large changes are followed by smaller ones. This suggests that the returns may not be independent and identically distributed (iid), as the time series plots reflect characteristics typical of financial markets, including high volatility and extreme movements.

4.2 Pearson correlation coefficient test results

Pearson correlation coefficient test is a well known statistical test that is used to measure the correlation between paired variables. Understanding that the paired variables might have some sort of a relationship is important for a study of this nature and results are presented in Table 5.

Table 5: Pearson correlation coefficient test results of BRICS returns

Parameter 1	Parameter 2	r	Correlation	Prob.
BVSP	IMOEX	0.3097	7.0400	<0.0000
BVSP	NSEI	0.4490	10.8600	<0.0000
BVSP	SSE	0.2124	4.7000	<0.0000
BVSP	JSE	0.3955	9.3100	<0.0000
IMOEX	NSEI	0.3395	7.8000	<0.0000
IMOEX	SSE	0.1742	3.8200	<0.0000
IMOEX	JSE	0.3010	6.9000	<0.0000
NSEI	SSE	0.1921	4.2300	<0.0000
NSEI	JSE	0.4439	10.7000	<0.0000
SSE	JSE	0.2750	6.1800	<0.0000

Note: 5% significant level is used.

Table 5 shows that BVSP and the NSEI have the strongest correlations with correlation values of 0.4490. Interestingly, China's correlation with the other four countries is low. The reason could be that the market reflects portfolio investments rather than trade flows, which is better represented by export and trade data, because it exhibits evidence of capital market flows (Dibua, 2020). The H_0 is rejected since all the $p - values$ are less than 5% level of significance. This indicates that there is a relationship between the BRICS returns. Furthermore, all the returns seem to be related to one another, which suggests that the BRICS countries may be connected. Correlation scatter plots are used to check the extent to which the variables are correlated. The correlation scatter plots and Pearson correlation coefficient test must have the same conclusion. Correlation scatter plots are shown in Figure 3.

Figure 3: Correlation scatter plots of the BRICS market returns

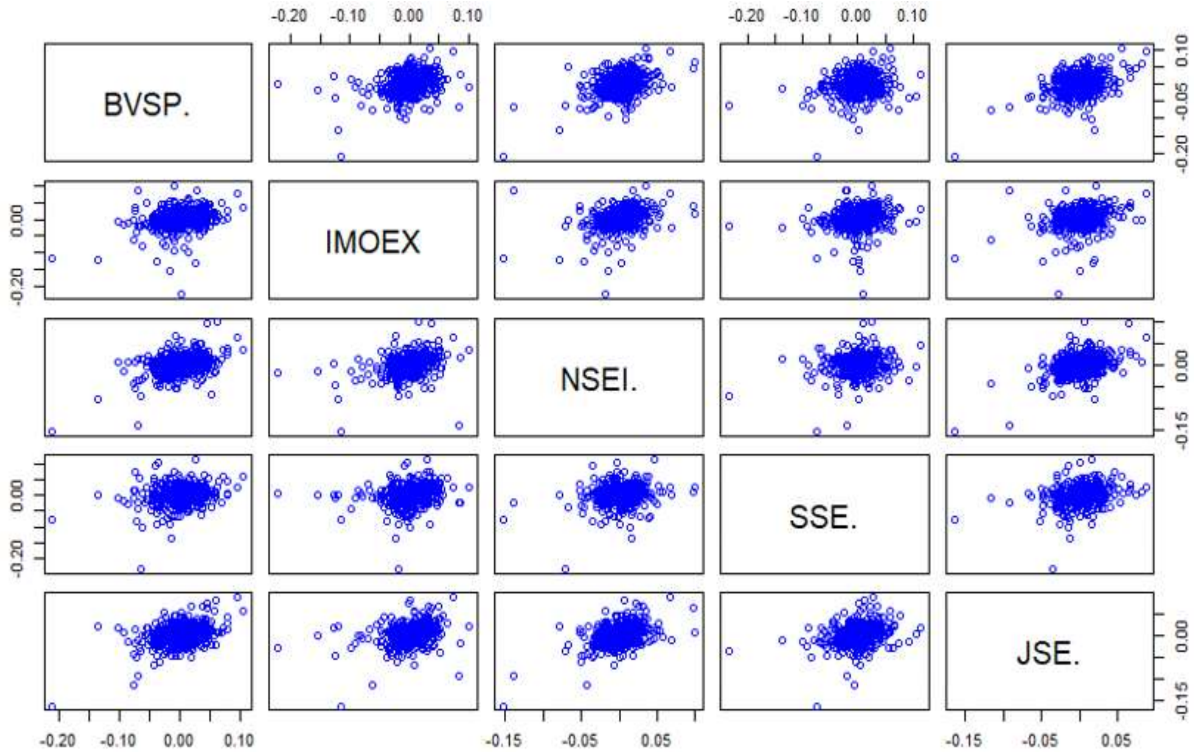


Figure 3 displays a weak uphill linear relationship (clustering in the corners) demonstrating the existence of tail dependence, hence the use of copula models in section 4.3.

4.3 Results of the ARMA model

Determining whether the returns exhibit serial correlation is essential. The Box-Ljung test statistic is used for this purpose. Additionally, this test helps identify the appropriate lag orders for an ARMA (p, q) model. The Box-Ljung test statistic values are provided in Table 5.

Table 5: Serial correlation test results of BRICS markets

Returns	Lags	Box-Ljung	Prob.
	1	0.1710	0.6792
	3	0.2671	0.8750
BVSP	3	0.9722	0.8080
	4	2.0765	0.7217
	5	2.8354	0.7253
	1	11.9410	<0.0000
IMOEX	1	0.1710	0.6792
	3	0.2671	0.8750
	3	0.9722	0.8080
	4	2.0765	0.7217
	1	0.0377	0.8459
NSEI	3	0.0379	0.9812
	3	0.5390	0.9102
	4	1.9069	0.7529
	5	2.9644	0.7055
	1	2.1118	0.1462
SSE	3	2.3502	0.3088
	3	2.7371	0.4340
	4	3.1887	0.5268
	5	3.2730	0.6580
	1	5.8344	0.0157
JSE	3	5.9659	0.0506
	3	6.7612	0.0799
	4	7.0750	0.1320
	5	7.7711	0.1693

Table 5 presents a summary of the serial correlation test results for BRICS market returns. The table indicates that the p-values for certain lag orders are below the 5% significance level, leading to the rejection of the null hypothesis (H_0). This suggests the presence of serial correlation at specific lags. While the test was expected to reveal some degree of correlation, the limited number of observations in weekly returns may not be sufficient to detect significant group serial correlation. Moreover, the absence of strong group serial correlation does not necessarily imply that returns are completely uncorrelated; rather, it may indicate that the serial correlation is not strong enough to be detected by the Box-Ljung test or similar statistical methods (Geweke, 1982). These findings are further supported by the ACF and PACF plots in Figures 3 and 4.

Figure 3: The ACF plots of BRICS returns

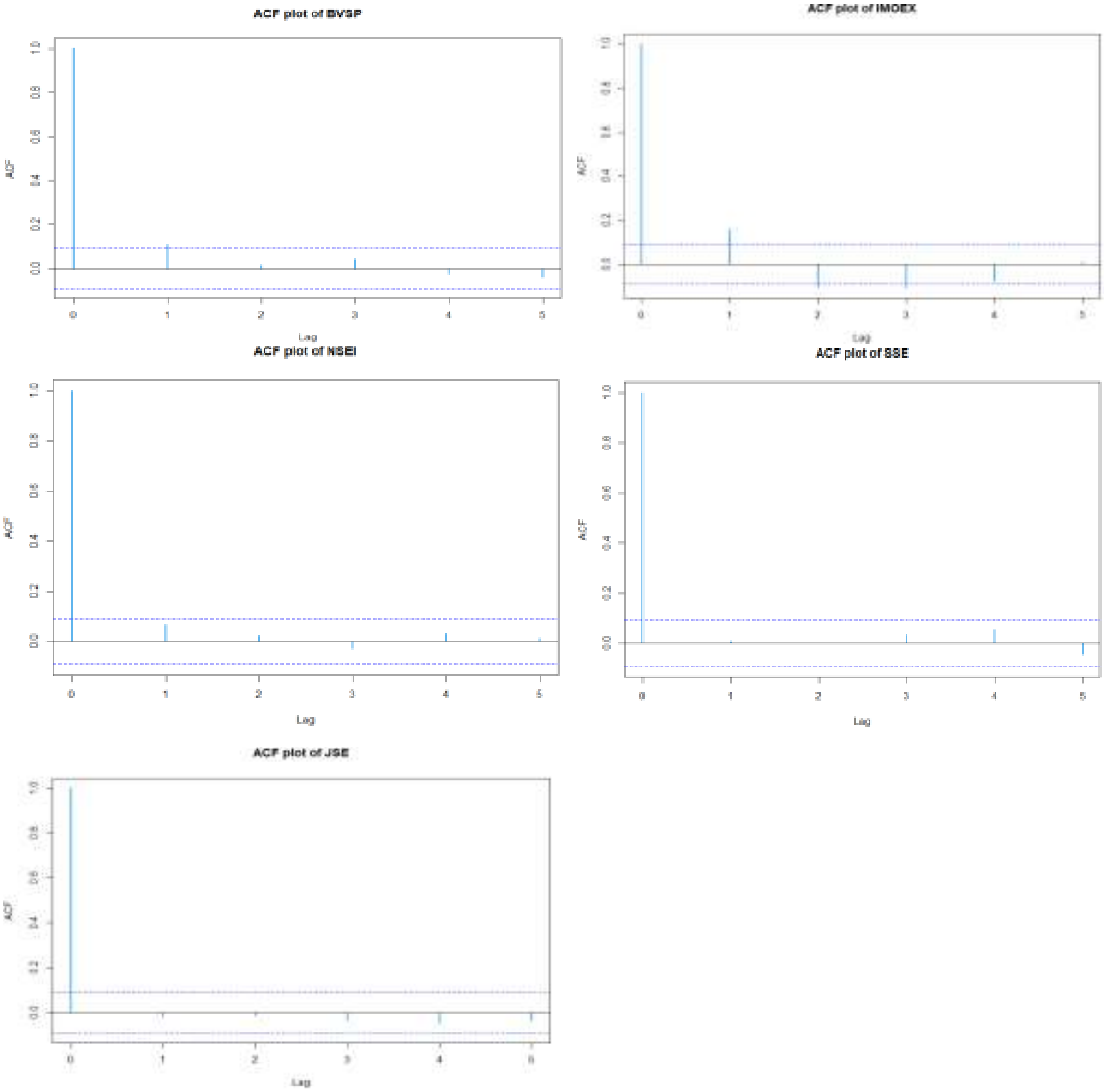
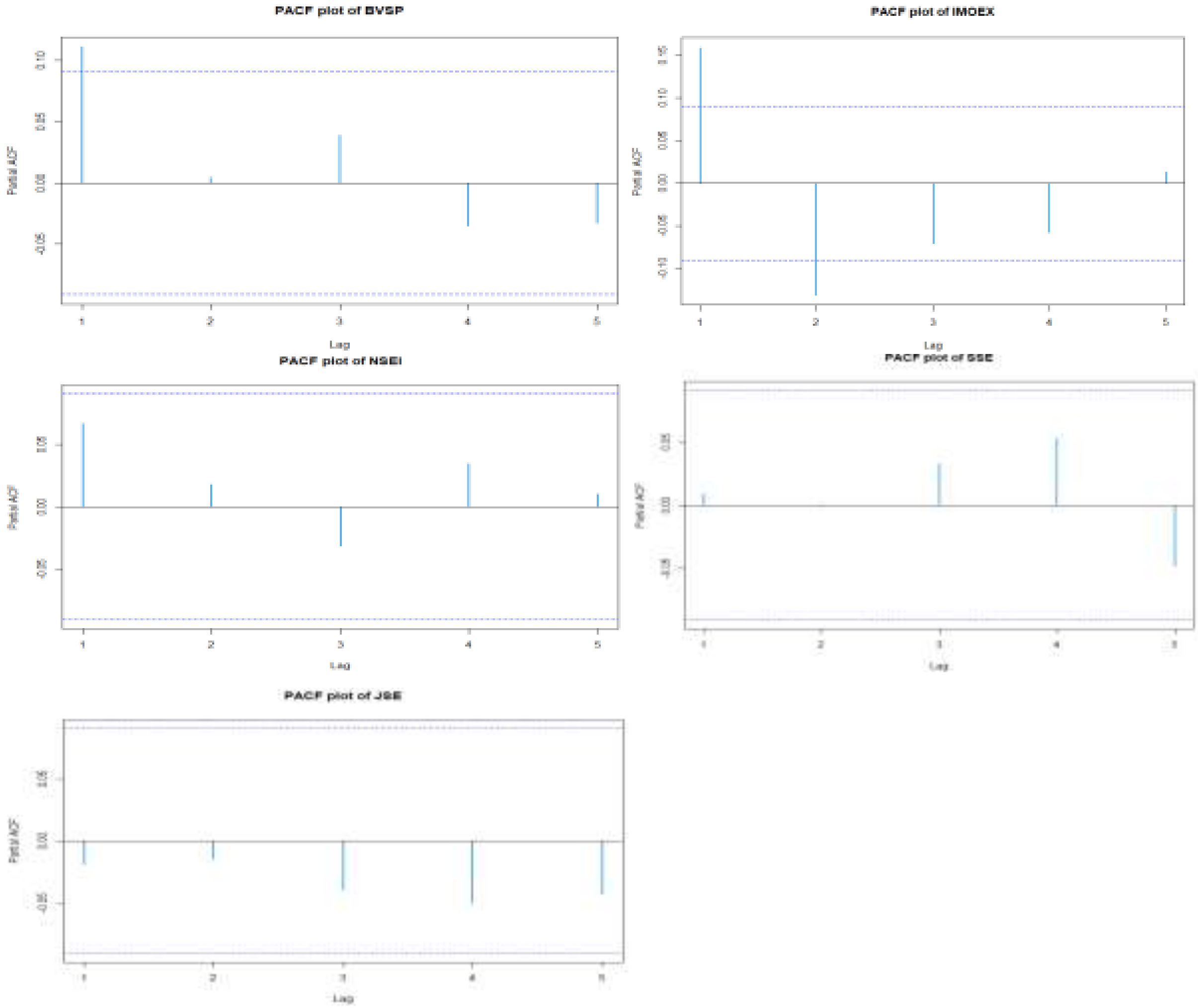


Figure 4: The PACF plots of BRICS returns



Figures 3 and 4 indicate that some lag orders are significant at the 95% confidence level. Figure 3 shows that all ACF plots follow a geometric pattern, where lag orders gradually decrease, while Figure 4 reveals that all PACF plots exhibit a cutoff. These patterns suggest that the MA (q) component of the ARMA (p, q) model is unnecessary, leading to a model with a lag order of $q = 0$. Including q in the model would complicate estimation. The lag order q would only be considered if both ACF and PACF displayed an exponential (geometric) decay pattern, as outlined by Rosadi (2012). However, Rosadi (2012) also noted that observing geometric patterns in both ACF and PACF is uncommon. Based on the visual inspection of the plots, the suggested models are ARMA $(1{,}0)$ for BVSP, ARMA $(1{,}0)$ for IMOEX, ARMA $(2{,}0)$ for NSEI, and ARMA $(3{,}0)$ for SSE, while the JSE plot does not indicate any significant lag orders. Nonetheless, model selection should not rely solely on visual inspection; other methods, such as the Information Criterion (Box *et al*., 2015), should also be considered. Model selection of the ARMA (p, q) models are presented in Table 6.

Table 6: Model selection of ARMA models for BRICS returns

Returns	Models	AIC
BVSP	ARMA(1,0)	**-1900.8952**
	ARMA(2,0)	-1825.2511
	ARMA(3,0)	-1752.2522
	ARMA(4,0)	1825.0215
	ARMA(5,0)	-1786.2255
IMOEX	ARMA(1,0)	-1856.3362
	ARMA(2,0)	1920.2525
	ARMA(3,0)	**-1950.2515**
	ARMA(4,0)	-1920.2215
	ARMA(5,0)	1900.2551
NSEI	ARMA(1,0)	**-2022.2515**
	ARMA(2,0)	-1925.2251
	ARMA(3,0)	-1985.2515
	ARMA(4,0)	-2019.2515
	ARMA(5,0)	-1825.3352
SSE	ARMA(1,0)	**-2021.2516**
	ARMA(2,0)	-1921.2516
	ARMA(3,0)	-1852.3251
	ARMA(4,0)	-1895.2250
	ARMA(5,0)	-2011.2516
JSE	ARMA(1,0)	**-2151.2255**
	ARMA(2,0)	-2140.2512
	ARMA(3,0)	-2131.2588
	ARMA(4,0)	-2150.9988
	ARMA(5,0)	-2100.2514

The competing models are summarised in Table 6, where the selection is based on the lowest Akaike Information Criterion (AIC) value. The results indicate that the chosen models are ARMA $(1,0)$ for BVSP, NSEI, SSE, and JSE, while the selected model for IMOEX is ARMA $(3,0)$. With the model selection complete, the next step is to proceed with parameter estimation. Estimated parameters of ARMA (p, q) models are shown in Table 7 to Table 11.

Table 7: Parameter estimation results of ARMA (1,0) model for BVSP

	ar1	Intercept
	0.2152	0.0011
Std Error	0.03529	0.0013

Table 8: Parameter estimation results of ARMA (3,0) model for IMOEX

	ar1	ar2	ar3	Intercept
	0.1526	-0.1215	-0.0875	0.0001
Std Error	0.03548	0.0512	0.0859	0.0012

Table 9: Parameter estimation results of ARMA (1,0) model for NSEI

	ar1	Intercept
	0.07859	0.0011
Std Error	0.03512	0.0022

Table 10: Parameter estimation results of ARMA (1,0) model for SSE

	ar1	Intercept
	0.0078	0.0008
Std Error	0.0354	0.0022

Table 11: Parameter estimation results of ARMA (1,0) model for JSE

	ar1	Intercept
	-0.0125	0.0011
Std Error	0.0351	0.0013

Tables 7 to 11 show that the standard errors of all estimated models are smaller than their respective coefficients, indicating statistical significance. However, for SSE and JSE, the standard errors are slightly larger than the coefficients. This does not necessarily imply that the chosen models are inappropriate. Hamilton (2020) noted that in certain cases, larger standard errors may be expected due to the complexity of the dataset, which can be influenced by irregular events such as economic recessions or natural disasters. Furthermore, it is essential to understand that the sign of the estimated ARMA (p, q) coefficients whether positive or negative is not the primary concern. Instead, their absolute values should be compared to the standard error values, as standard errors reflect the degree of uncertainty or variability in the estimates (Gujarati and Porter, 2009). Following parameter estimation, it is crucial to assess whether the estimated models conform to statistical assumptions. Diagnostic test results of the estimated ARMA (p, q) models are presented in Table 12.

Table 12: Box-Ljung results of ARMA models for BRICS fitted residuals

Returns	Model	Box-Ljung	Prob.
BVSP	ARMA (1,0)	5.5787	0.3494
IMOEX	ARMA (3,0)	2.8475	0.7235
NSEI	ARMA (1,0)	4.3789	0.4963
SSE	ARMA (1,0)	1.7956	0.8766
JSE	ARMA (1,0)	1.7956	0.8766

Table 12 displays that the BRICS returns have no serial correlation in the fitted residuals. The table further shows that the $p - values$ of the Box-LJung test statistics are all greater than 5% significance level making the null hypotheses (H_0) not to be rejected. Therefore, it can be concluded that there is no serial correlation in the fitted residuals. Therefore, all the ARMA (p, q) models selected are the relevent models to be used in Hybridisation processess. Model selection of the Hybrid ARMA (p, q)-EGARCH (1,1) models are presented in Section 4.4.

4.4 Results of the Hybrid ARMA (p, q)-EGARCH (1,1) models

The models to be selected, estimated, and diagnosed are computed using SSTD and GED. These conditional distributions are involved because of the characteristics of the BRICS returns. The BRICS returns are leptokurtic. Additionally, Ijumba (2013) mentioned the importance of involving distributions that reflect on the features of the observed time series processes. This procedure does not only assist with errors associated with non-Gaussian, but it allows risk managers and investors to make more informed decisions regarding the portfolio of an asset by providing better Value-at-Risk (VaR) and Expected Shortfall (ES) portfolio estimates. The results are summarised in Table 13.

Table 13: Model selection results of the Hybrid ARMA (p, q)-EGARCH (1,1) models for BRICS returns

Returns	Models	AIC	
		SSTD	**GED**
BVSP	ARMA(1,0)-EGARCH(1,1)	**-4.1698**	-4.1625
IMOEX	ARMA(3,0)- EGARCH(1,1)	**-4.5075**	-4.4702
NSEI	ARMA(1,0)- EGARCH(1,1)	**-4.9148**	-4.8859
SSE	ARMA(1,0)- EGARCH(1,1)	**-4.5212**	-4.4864
JSE	ARMA(1,0)- EGARCH(1,1)	**-4.8647**	-4.8320

The results in Table 13 suggest that the Hybrid ARMA (p, q)-EGARCH (1,1) models under SSTD are the best chosen models with the lowest AIC value. Table 14 to Table 18 summarises the parameter estimation of the selected models.

Table 14: Parameter estimation results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for BVSP

	Estimate	**Std. Error**	**t value**	**Prob.**
ar1	-0.3319	0.0459	-7.2309	<0.0000
Omega	-0.3229	0.0045	-71.1224	<0.0000
alpha1	-0.1289	0.0315	-4.0850	<0.0000
beta1	0.9528	0.0007	1361.8771	<0.0000
gamma1	0.0863	0.0196	4.4158	<0.0000
Skew	0.8344	0.059	14.1073	<0.0000
Shape	8.6671	3.0724	2.8209	0.0048

Table 15: Parameter estimation results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for IMOEX

	Estimate	**Std. Error**	**t value**	**Prob.**
ar1	0.0429	0.0478	0.8977	0.3694
ar2	-0.1004	0.0476	-2.1093	0.0349
ar3	-0.0421	0.0446	-0.9446	0.3449
Omega	-0.7625	0.3253	-2.3437	0.0191
alpha1	-0.1672	0.0515	-3.2486	0.0012
beta1	0.8935	0.0452	19.7892	<0.0000
gamma1	0.1949	0.0767	3.5421	0.0020
Skew	0.7778	0.0538	14.4695	<0.0000
Shape	6.5774	1.9160	3.4330	0.0006

Table 16: Parameter estimation results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for NSEI

	Estimate	Std. Error	t value	Prob.
ar1	0.3456	0.0528	6.5456	<0.0000
Omega	-0.5730	0.0856	-6.6957	<0.0000
alpha1	-0.1294	0.0307	-4.2149	<0.0000
beta1	0.9228	0.0116	79.7614	<0.0000
gamma1	-0.1224	0.0481	-3.5462	<0.0000
Skew	0.7846	0.0476	16.4669	<0.0000
Shape	6.0860	1.7913	3.3974	<0.0007

Table 17: Parameter estimation results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for SSE

	Estimate	Std. Error	t value	Prob.
ar1	-0.6290	0.0444	-14.1667	<0.0000
Omega	-0.2052	0.0932	-2.1723	0.0298
alpha1	-0.0305	0.0050	-6.1000	<0.0000
beta1	0.8717	0.0129	67.5733	<0.0000
gamma1	0.2717	0.0642	3.2355	<0.0000
Skew	0.8385	0.0509	16.5045	<0.0000
Shape	4.6884	1.0464	4.4802	<0.0000

Table 18: Parameter estimation results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for JSE

	Estimate	Std. Error	t value	Prob.
ar1	0.4560	0.0528	8.6364	<0.0000
Omega	-1.0259	1.4634	-0.7011	0.4833
alpha1	-0.2590	0.0986	-2.6250	0.0087
beta1	0.8654	0.1923	4.5004	<0.0000
gamma1	-0.1244	0.0633	-3.9643	<0.0000
Skew	0.8300	0.0474	17.5026	<0.0000
Shape	7.20890	0.8485	8.4963	<0.0000

Tables 14 to 18 shows that the parameter α_1 and β_1 are significant demonstrate that the Hybrid ARMA (p, q)-EGARCH $(1,1)$ model effectively captured volatility clusters. Additionally, (P) is greater than 0.9 but less than 1, implying that leverage effect is present. The high value of leverage effect, suggest that the volatility shocks are long-lasting meaning that, if a market experiences high volatility, it is likely to continue experiencing high volatility for some time (Nasir *et al.*, 2023). A Sign Bias test was computed and the results revealed that the Hybrid ARMA (p, q)-EGARCH $(1,1)$ model is suitable for capturing the leverage effect on all the countries. All the countries have statistically significant $p - values$ (See Appendix A1 to A5).

Table 19: The LM-ARCH and JB results of the Hybrid ARMA (p, q)-EGARCH (1,1) models for the BRICS fitted squared standardised residuals

Returns	Models	LM-ARCH	Prob.	JB	Prob.
BVSP	ARMA(1,0)-EGARCH(1,1)	2.7350	0.5649	73.1670	<0.0000
IMOEX	ARMA(3,0)-EGARCH(1,1)	2.7394	0.2351	123.0735	<0.0000
NSEI	ARMA(1,0)-EGARCH(1,1)	1.8464	0.7498	68.5664	<0.0000
SSE	ARMA(1,0)-EGARCH(1,1)	4.2655	0.3099	176.7664	<0.0000
JSE	ARMA(1,0)-EGARCH(1,1)	0.9936	0.9146	125.4932	<0.0000

The results of the LM-ARCH test statistics summarised in Tables 19 indicate that the fitted squared standardised residuals exhibit no ARCH effects, as all $p - values$ exceed the 5% significance level, leading to the acceptance of H_0. Consequently, all the estimated models have successfully passed the diagnostic test, confirming their suitability for volatility forecasting and VaR measurement. Additionally, the JB test statistics for all estimated models reveal p-values below the 5% significance level, resulting in the rejection of H_0. This implies that the fitted squared standardised residuals are not normally distributed. However, it is important to note that several studies have highlighted the JB test's limitations when applied to heteroscedastic models, suggesting that alternative normality tests may be more appropriate, as recommended by Ijumba (2013).

4.5 The results of the copula models

The purpose of the copula model is to determine the behavior of the dependence structure between the markets. Before fitting copula models, it is important to extract the residuals from all the Hybrid ARMA (p, q)-EGARCH (1,1) models estimated under SSTD. After the process of extracting the residuals from the Hybrid ARMA (p, q)-EGARCH (1,1) model is done, then the conditional distributions of the extracted residuals are transformed into a uniform distribution, which will then be used to fit the copula models. This transformation is attractive because it supports the construction of a multivariate distribution function (Cerrato *et al.*, 2015). Model selection results of copula models for BRICS returns are summarised in Table 20.

Table 20: Model selection results of copula models for BRICS returns

Paired returns		Student-t	Clayton	Frank	Gumbel
JSE-BVSP	LL	-315.9780	-289.8520	-263.5240	-300.5740
	AIC	-631.9560	-579.7030	-527.0480	-601.1490
JSE-IMOEX	LL	-242.01900	-228.2060	-200.0620	-234.4590
	AIC	-484.038	-456.4170	-400.1250	-468.9170
JSE-NSEI	LL	-2063.310	-1789.828	1826.337	-2001.391
	AIC	-4126.5820	-3579.6730	-360.7490	-4002.7390
JSE-SSE	LL	-137.8760	-94.110	-84.9180	-96.8070
	AIC	-275.7520	-188.221	-169.8360	-193.6140
BVSP-IMOEX	LL	-780.4610	-656.4030	-634.4370	-701.8820
	AIC	-1560.7720	-1311.9940	-1268.9830	-1403.7210
BVSP-NSEI	LL	-827.8080	-705.3780	-683.1370	-768.7910
	AIC	-1655.5230	-1410.6900	-1366.1330	-1537.5460
BVSP-SSE	LL	-379.677	-365.2560	-326.0680	-377.2510

Paired returns		**Student-t**	**Clayton**	**Frank**	**Gumbel**
	AIC	-759.3544	-730.5130	-652.1350	-754.5030
IMOEX-NSE	LL	962.9000	-859.5000	-838.6340	-912.5110
	AIC	-1925.7090	-1719.663	-1677.3110	-1824.9700
IMOEX-SSE	LL	-772.2190	-683.7530	-668.0390	-726.1000
	AIC	-1544.4430	-1367.5540	-1336.1550	-1452.2230
NSEI-SSE	LL	-319.6440	-290.5830	260.3880	-290.9380
	AIC	-639.2890	-581.1660	-520.7770	-581.8770

Table 20 show that the Student-t copula gives the best fit since it has the highest LL and lowest AIC, followed by Gumbel and then Clayton. It should be noted that the two dependence parameters of the Student-t copula have the same LL and AIC values, hence only one value is provided. Since the models have been selected. Estimated parameters of the copula models are shown in Table 21.

Table 21: Parameter estimation results of copula models for BRICS markets

Paired returns		**Student-t**	**Clayton**	**Frank**	**Gumbel**
JSE-BVSP	Estimate	0.6076	0.3898	2.5130	1.3291
	$p-value$	0.0286	0.0134	0.0525	0.0173
JSE-IMOEX	Estimate	1.2745	0.5101	2.1566	0.3489
	$p-value$	0.0162	0.0140	0.0478	0.0273
JSE-NSEI	Estimates	8.4078	2.3618	0.8311	2.5486
	$p-value$	0.1378	0.0519	0.0040	0.0356
JSE-SSE	Estimates	1.3964	0.2283	1.1813	0.2904
	$p-value$	0.0383	0.0156	0.0143	0.0243
BVSP-IMOEX	Estimates	0.5737	1.0011	4.2170	1.6605
	$p-value$	0.0027	0.0767	0.0034	0.0099
BVSP-NSEI	Estimates	0.5951	1.0484	4.3955	1.6912
	$p-value$	0.0231	0.0794	0.0343	0.0094
BVSP-SSE	Estimates	1.3798	0.6683	2.7906	0.4379
	$p-value$	0.0180	0.0292	0.0563	0.0125
IMOEX-NSE	Estimates	0.6482	1.2383	4.8005	1.7387
	$p-value$	0.0082	0.0362	0.0846	0.0236
IMOEX-SSE	Estimates	0.5960	1.0337	4.1712	1.6186
	$p-value$	0.0094	0.0336	0.0756	0.0217
NSEI-SSE	Estimates	0.3866	0.5801	2.4685	1.3319
	$p-value$	0.0173	0.0134	0.0524	0.0284

Table 21 shows that the dependence parameter for all the copula models is higher indicating greater dependence structure between the paired countries. This means that the countries move together either in an upward move or downward move. Furthermore, the results suggest that there is an asymmetric tail dependence between the markets. An asymmetric tail dependence is when the market moves more together either in an upward trend or a downward trend. Lastly, Student-t copula better explains the dependence structure between the markets while Gumbel and Clayton better explain the relationship between the markets.

This conclusion suggests that the copula models utilised are well fit for the BRICS markets. Since there is evidence of tail dependence, it is important that tail distribution estimation is included in the current study. Estimated tail distribution of the copula models are summarised in the Table 22.

Table 22: Estimates of the left and upper tail dependence coefficients

	Student-t	**Clayton γ_L**	**Frank**	**Gumbel γ_U**
JSE-BVSP	0.0000	0.3301	0.0000	0.3033
JSE -IMOEX	0.0000	0.4710	0.0000	0.5105
JSE -NSEI	0.0000	0.5222	0.0000	0.5899
JSE -SSE	0.0000	0.3211	0.0000	0.3421
BVSP-IMOEX	0.0000	0.5266	0.0000	0.5001
BVSP -NSEI	0.0000	0.4999	0.0000	0.4910
BVSP -SSE	0.0000	0.0899	0.0000	0.2325
IMOEX -NSEI	0.0000	0.6989	0.0000	0.2354
IMOEX-SSE	0.0000	0.2814	0.0000	0.2601
NSEI-SSE	0.0000	0.3256	0.0000	0.3215

Table 22 display that JSE with BVSP, IMOEX, NSEI, and SSE have a lower tail dependence, which suggests an asymmetric lower tail dependence structure between these paired markets. The lower tail dependence between these paired countries is captured by Clayton copula. The results indicates that the South African market together with Brazil, Russia, India, and China co-move as one. Similar to JSE with other BRICS markets, there is evidence of upper tail dependence structure between, BVSP with IMOEX, NSEI, and SSE. The results suggest an asymmetric upper tail dependence structure between these countries. The upper tail dependence between these countries is captured by the Gumbel copula model. The rest of the paired countries are captured by the Gumbel copula model. Lastly, the results suggest that the paired countries that have been captured by the Clayton copula model move together more when the markets go down than when the markets go up, and vice versa with the paired countries that have been captured by the Gumbel copula model.

5. Conclusion

The study modelled volatility clusters, leverage effects and the dependency structure among BRICS market returns using a Copula ARMA (p,q)-EGARCH (1,1) approach. Both the ADF and PP tests indicated that the BRICS closing prices are not stationary at level; however, they became stationary after first differencing. The findings from the Pearson correlation coefficient test showed that all paired variables have a positive linear relationship and are less than 0.5, which is a sign of tail dependency.The results suggests that that the returns move together in the same direction. Furthermore, the correlation scatter plots also displayed an upward-sloping positive linear relationship. All the plots showed signs of tail dependence, making it important to employ the copula model, which can assess both lower and upper tails of the distribution.

Diagnostic tests revealed that all the models were adequate. These results were further supported by the ACF plots, which showed that all lags were insignificant, indicating that these models are appropriate. The study further introduced the Hybrid ARMA (p,q)-EGARCH (1,1) model. These models succesfully captured leverage effect. The models also passed diagnostic tests, indicating that it is appropriate for use in volatility forecasting and Value-at-Risk (VaR) estimation. These findings are consistent with those of Yaziz *et al.* (2013) and Tan *et al.* (2010).

Residuals extracted from the Hybrid ARMA (p,q)-EGARCH (1,1) models were used to compute the copula models. These conditional residuals were transformed into a uniform distribution, which was then used to fit the copula models. The study employed Student-t, Frank, Clayton, and Gumbel copula models to describe the dependence structure. The findings suggested that the Student-t copula model was the best fit, based on the lowest AIC value. Similar results were evident from the study by Charfeddine and Benlagha (2016). The dependence parameters of all copula models were statistically significant for each country pair, indicating a meaningful relationship in terms of market behavior, potentially reflecting trade linkages. The estimates from the Clayton copula model showed that JSE's dependence with BVSP, IMOEX, NSEI, and SSE exhibited lower tail dependence, suggesting that these markets tend to move together more during downturns. Similarly, the Gumbel copula model captured upper tail dependence between BVSP and IMOEX, NSEI, and SSE, indicating that these markets move together more during upturns. The remaining country pairs were also best described by the Gumbel copula model. These findings are consistent with the theories proposed by Trang (2021) and Dibua (2020).

The study highlights significant financial interconnectedness and tail dependence among BRICS markets, indicating that these markets often move in the same direction, particularly during periods of extreme volatility. This reduces the effectiveness of diversification strategies and raises the likelihood of contagion during market downturns. The observed asymmetric tail dependence and leverage effects underscore the necessity for strong risk management practices and effective regulatory measures. The use of copula-based models, especially the Student-t and Joe-Clayton copulas, enhances the accuracy of risk assessment and portfolio optimisation. Overall, the results stress the value of coordinated financial strategies and regional collaboration to strengthen the stability and resilience of BRICS financial systems.

Future research could expand on this study by incorporating more flexible distributions, such as the Normal Inverse Gaussian (NIG) distribution. This distribution can better capture the shape, skewness, and scale of data by adjusting its parameters. Using such a versatile distribution allows for more accurate modeling of innovations. Additionally, future studies could consider employing Joe-Clayton (JC) copulas to model the dependency structure among BRICS markets. The JC copula uses the Christoffersen test to estimate portfolio VaR, which plays a critical role in regulating capital requirements for potential investors.

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Appendix

Table A1: The Sign Bias test results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for BVSP

	Estimate	Std.Error	t value	Prob.
(Intercept)	0.8831	0.1869	4.7260	<0.0000
x_1	0.3520	0.2554	1.3790	0.1690
x_2	-23.3789	10.1892	-2.2945	0.1526
x_3	-0.4734	6.1698	-0.0770	0.9390

Table A2: The Sign Bias test results of the Hybrid ARMA (3,0)-EGARCH (1,1) model for IMOEX

	Estimate	Std.Error	t value	Prob.
(Intercept)	0.9385	0.1910	4.9120	<0.0000
x_1	0.1361	0.2534	0.5370	0.5915
x_2	-10.0062	5.7913	-1.7278	0.3845
x_3	-1.1430	7.6067	-0.1500	0.8806

Table A3: The Sign Bias test results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for NSEI

	Estimate	Std.Error	t value	Prob.
(Intercept)	0.9030	0.1519	5.9430	<0.0000
x_1	-0.1429	0.2178	-0.6560	0.5119
x_2	11.3536	5.9947	1.8939	0.1256
x_3	9.4749	7.0649	1.3410	0.1805

Table A4: The Sign Bias test results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for SSE

	Estimate	Std.Error	t value	Prob.
(Intercept)	1.0430	0.1872	5.573	<0.0000
x_1	0.1242	0.2657	0.4670	0.6400
x_2	-6.1707	5.4520	-1.1320	0.2580
x_3	-10.3398	7.0286	-1.4710	0.1420

Table A5: The Sign Bias test results of the Hybrid ARMA (1,0)-EGARCH (1,1) model for JSE

	Estimate	Std.Error	t value	Prob.
(Intercept)	1.3530	0.1865	7.2560	<0.0000
x_1	-0.4298	0.2750	-1.5630	0.1188
x_2	0.8258	7.5923	0.1090	0.9134
x_3	12.9294	8.6866	1.4884	0.2980