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HEURISTIC AND EQUILIBRIUM STRATEGIES IN PREMIUM AUCTIONS

Abstract:

Premium auctions are conducted in two stages. In the first stage bidders compete in English auction until two bidders remain. The two finalists enter the second stage where they compete in a first-price sealed bid auction (Amsterdam auction) or in another English auction (Antwerp auction). The winner and the runner up obtain the premium that is proportional to the difference between the runner up's bid and the highest losing bid in the first stage. We compare the equilibrium bidding strategy of the two finalists with the three heuristic strategies when bidders have private values in Amsterdam auction. With the first heuristic strategy, each finalist believes that he will lose in the second stage and that his bid determines the amount of the premium. With the second heuristic strategy, each finalist is optimistic and has second-order belief that the other finalist is pessimistic. With the third heuristic strategy, both finalists are optimistic and have second-order belief that the other finalist is optimistic.
The simulation analysis with symmetric bidders shows that the average increase of bid of the runner up relative to the equilibrium bid is the largest with the second heuristic strategy, followed by the third and the first, respectively. The premium with either heuristic strategy is larger than with the equilibrium strategy. The profit of the winner is larger than the equilibrium profit, while seller's revenue is lower than in equilibrium. The same conclusions hold for strongly asymmetric bidders. The use of heuristic strategies could be considered as a form of tacit collusion between two finalists, and with symmetric bidders who use the first heuristic strategy, the tacit collusion is stable even in a one-shot game.

Keywords:

Hybrid auctions, Amsterdam auction, Antwerp auction, perfect Bayesian equilibrium, heuristic strategies

JEL Classification: D44

Introduction

Auction theory was developed by William Vickrey (1961) as a form of static game of imperfect information. According to Harshanyi (1967) doctrine, in these games Nature makes the first move and determines the type of each player. In auction games, players are bidders who bid for the object that seller wants to sell. Bidder's type is his value, i.e. his reservation price (the maximal amount he is willing to pay for the object). Bidder's strategy is to submit a bid that depends on his value. This value is drawn from some probability distribution (typically uniform) and is known only to the bidder. His rivals know their own values and know only the probability distribution of other bidders' values. Values can be private when random variables representing values are independent or interdependent when the value of one bidder is in positive correlation with the value of other bidders. In fact, with interdependent values each bidder obtains a signal (also a random variable) about the true value of the object that is unknown to bidders. The true value is a function of all signals. Since bidder bids based on his signal and does not know the true value, the winner's curse can occur when the winner pays more than the true value. The distinction between the two types of values is supported by the well-known distinction in game theory between independent and correlated types of players.

Auctions could be also considered as a form of mechanism design, Myerson (1981). In the mechanism design approach, it is necessary to construct a set of rules such that each bidder finds it optimal to submit a bid that is equal to his value. The mechanism with this property is called incentive compatible mechanism.

The simple auction forms can be divided in open and closed auctions. In open auctions bidders compete in open outcry competition for the object. Two open auction forms are English and Dutch auction. In English auction, the auctioneer starts the auction with a low price, and as he raises the price bidders drop out from the auction one by one. The last active bidder wins the auction and pays the price at which next-to-last bidder dropped out. In Dutch auction, the auctioneer starts the auction with a high price and reduces it gradually until the first bidder accepts that price and pays the object at the current price. In sealed bid auctions bidders submit bids in sealed envelopes. In a first-price auction, the bidder with the highest price wins the object and pays his bid. In a second-price auction, the winner pays the second highest bid.

When bidders have private values, English and second-price auctions are equivalent and each bidder bids his value since the amount that he pays depends on the runner up's bid. These two mechanisms are incentive compatible. In a first-price and Dutch auction, bidder bids lower than his value to obtain a certain surplus if he wins and these two auctions are also strategically equivalent, but are not incentive compatible.

Each auction mechanism has to accomplish two objectives. The first is to maximize seller's expected revenue and the second one is to allocate the object to the bidder with the highest value (efficiency). In some auction forms there is a conflict between the two objectives. In order to mitigate drawbacks of simple auction forms that fail to achieve one of the objectives, simple auctions are combined in two stage hybrid auctions and premium auctions.

In this paper we will analyze two premium auctions, Amsterdam and Antwerp auction. We will compare the equilibrium strategies with some heuristic strategies that bidders may use in Amsterdam auction. The rest of the paper is organized as follows. In the second part we describe premium auctions and equilibrium bidding strategies. In the third part, we derive heuristic bidding strategies in Amsterdam auction and in the following section present the results of the simulation analysis with these strategies. The last section is reserved for concluding remarks.

Premium auctions

Hybrid auctions consist of two stages. In the first stage bidders participate in one type of auction and in the second stage, they participate in another type of auction. In the Anglo-Dutch auction proposed by Klemperer (1998), bidders first participate in an English auction until two bidders remain. The two finalists enter the second stage where they submit sealed-bids in a first-price auction that should be higher than the price at which the first stage was finished. The winner is the bidder with the highest bid who pays its bid. In the Dutch-Anglo auction proposed by Dutra and Menezes (2002) as well as Levin and Ye (2008), bidders first participate in a sealed bid first-price auction. If the highest bid in the first stage is sufficiently higher that the second highest bid, there is no second stage and the auction is over. Otherwise, all bidders who have submitted bids sufficiently close to the highest bid enter the second stage where they compete in an English auction with the starting price equal to the highest bid in the first stage. The purpose of combining simple auctions in hybrid forms is to mitigate drawbacks of some simple auctions and use their advantages. Hybrid auctions increase the level of competition among bidders and reduce the risk of collusion.

Premium auctions also contain two stages, but the two finalists receive a premium from the seller. In Amsterdam auction proposed by Goeree and Offerman (2004), there is an English auction in the first stage until two bidders remain. The price at which the last bidder dropped out from the first stage auction is called the bottom price. The two finalists submit sealed bids in the second stage. In the first-price Amsterdam auction the winner of the second stage pays his bid, while in the second-price Amsterdam auction, the winner of the second stage pays the runner up's bid. In either type of Amsterdam auction both finalists receive a premium that is some proportion, $\alpha \in (0,1/2]$, of the difference between the runner up's bid and the bottom price. This premium motivates the runner up to bid higher and force the winner to pay a higher price.

In the second-price Amsterdam auction, Goeree and Offerman (2004) prove that the bidding strategy of bidder with value *v* that is uniformly distributed on the interval [0,100] is the same in both stages and equal to:

$$b_1(v) = b_2(v) = 100\frac{\alpha}{(1+\alpha)} + \frac{v}{(1+\alpha)} . \tag{1}$$

Goeree and Offerman (2004) suppose that bidders could be symmetric, weakly asymmetric or strongly asymmetric. In the symmetric case, bidders have uniformly distributed values over the same interval. In the weakly asymmetric case, there is one strong bidder whose value is uniformly distributed on the interval [*L*,*H*], for 0<*L*<100 and *n*-1 weak bidders with values uniformly distributed on the interval [0,100]. In the strongly asymmetric case there is no overlapping region of the supports of the probability distributions of weak and strong bidders and in that case *L*>100. In the experimental part of the paper, Goeree and Offerman (2004) show that in the weakly asymmetric case seller obtains the highest revenue with the first-price auction, followed by

Amsterdam auction and English auction. In the strongly asymmetric case, Amsterdam auction yields the highest revenue for the seller followed by first-price and English auctions.

In the real experiment conducted by Onderstal (2017) it is shown that with symmetric bidders English and second-price Amsterdam auction generate the same expected revenue to the seller, but the variance of expected revenue is higher in English auction.

When cartel is formed in an auction, the members of the cartel elect the cartel's representative who will bid at the main auction, while other cartel members will not participate in the main auction. Cartel reduces the competition at the main auction enabling its representative to pay lower price for the object at the main auction. When bidders are symmetric, Hu, Offerman and Onderstal (2011) have proved that cartel is less stable in Amsterdam and first-price auction than in English auction. With asymmetric bidders, the probability of cartel formation is the lowest in Amsterdam auction among the 3 considered auctions.

Antwerp auction has the same rules as Amsterdam auction, except that in the second stage there is another English auction, as described by Hu, Offerman and Zou (2011). The premium for two finalists is determined in the same fashion as in Amsterdam auction. When bidders have private values, Antwerp auction is equivalent to a second-price Amsterdam auction. In the private value case, Hu, Offerman and Zou (2018) have shown that if bidders are risk neutral, the expected revenue of the seller is the same in Antwerp and English auction, but the variance of revenue is higher in English auction. This means that a risk averse seller prefers Antwerp auction to English auction.

Concerning the bidding strategy in Antwerp auction, Brunner, Hu and Oechssler (2014) have shown that the bidding strategy is a decreasing function of bidder's risk aversion. Nevertheless, even risk averse bidder bids higher than value. Risk averse bidder bids less aggressively than risk neutral bidder and risk seeking bidder bids more aggressively than risk neutral bidder. This implies that the expected revenue of the seller is larger (lower) in Antwerp auction than in English auction when bidders are risk seeking (risk averse).

In the experimental part of the paper Brunner, Hu and Oechssler (2014) have determined that the actual strategy of bidders departs from equilibrium strategy in Antwerp auction and their explanation is that bidders might have used heuristic strategies. They have identified 4 possible heuristic strategies. First, a bidder stays active until the price reaches its value. Second, a bidder stays active until the price reaches the sum of its value and current premium (the difference between the current price and the bottom price). Third, a bidder may leave the auction when two bidders remain (out of 4 bidders at the beginning of the experiment). Fourth, a bidder may leave the auction as soon as the first bidder leaves it. Bidder's profit is only marginally lower than the profit with the equilibrium strategy for the first two heuristic strategies and considerably lower for the last two heuristic strategies.

Heuristic strategies in Amsterdam auction

Sometimes it is quite complicated for bidders to calculate optimal strategy, which is certainly the case with premium auctions. Therefore, it is more likely that bidders in premium auctions rely on certain rules of thumb in determining their bid. When bidders have private values, but use heuristic strategies, Amsterdam and Antwerp auctions are no longer equivalent.

We have described previously 4 heuristic strategies analysed by Brunner, Hu and Oechssler (2014) in Antwerp auction. In this paper, we will consider 3 possible heuristic strategies for Amsterdam auction and compare them with the equilibrium strategy. The main difference between Antwerp and Amsterdam auction is that in the former a bidder can calculate the current premium in the second stage at any moment, while in the latter this is not possible since bids are submitted in sealed envelopes. Therefore, heuristic strategies in Amsterdam auction are more complex and include first order belief of the bidder whether his bid will be the losing bid in the second stage that determines the amount of the premium, or it will be the opponent’s bid. The heuristic strategy is also based on the second-order belief of a bidder about the belief of his opponent.

We will consider second-price Amsterdam auction when bidders have private values. In the first stage, when English auction is used, we assume that bidders follow the heuristic strategy of leaving the auction when the price reaches their value. When the first stage is completed, two finalists enter the second stage and they can observe the price at which the last bidder dropped out from the first stage (bottom price). We will denote the bottom price by $v^{(3)}$.

In the second stage, the bidder can obtain a premium whether he wins or loses the auction. Therefore, in the second stage, bidder can bid higher than his value. However, the question is how he will determine this amount by which his bid is higher than his value and the 3 various assumptions about his beliefs differentiate our 3 heuristic strategies.

For the first heuristic strategy, we assume that bidder has pessimistic belief that he will lose in the second stage and that his bid determines the amount of the premium. Consider bidder 1 with the highest value and such belief. His bid is equal to the sum of his value and the premium:

$$b^{(1)} = v^{(1)} + \alpha[b^{(1)} - v^{(3)}], \tag{2}$$

$$b^{(1)} = \frac{v^{(1)} - \alpha v^{(3)}}{1-\alpha}. \tag{3}$$

In the same fashion we can determine the bid of bidder with the second highest value:

$$b^{(2)} = \frac{v^{(2)} - \alpha v^{(3)}}{1-\alpha}. \tag{4}$$

The second assumption about bidders’ beliefs is that one bidder has optimistic beliefs and believes that he will win the auction and that the amount of the premium will be determined by the other finalist’s bid and his second order belief is that the other finalist believes that he will lose in the second stage. Therefore, the bid of bidder 1 with the highest value is:

$$b^{(1)} = v^{(1)} + \alpha[b^{(2)} - v^{(3)}] = v^{(1)} + \alpha\left[\frac{E[v^{(2)}] - \alpha v^{(3)}}{1-\alpha} - v^{(3)}\right] = v^{(1)} + \frac{\alpha}{1-\alpha}\left[E[v^{(2)}] - v^{(3)}\right], \tag{5}$$

where the second equality is obtained by substituting (4) in accordance with the second order belief and the expectations term stems from the fact that bidder 1 does not know the value of his opponent. The heuristic strategy of bidder 2 is symmetric:

$$b^{(2)} = v^{(2)} + \frac{\alpha}{1-\alpha}\left[E[v^{(1)}] - v^{(3)}\right]. \tag{6}$$

The last heuristic strategy is based on the assumption that both bidders have optimistic beliefs and believe that the other bidder's bid will determine the amount of the premium. In this case we have the following system of strategies:

$$b^{(1)} = v^{(1)} + \alpha[b^{(2)} - v^{(3)}], \tag{7}$$

$$b^{(2)} = v^{(2)} + \alpha[b^{(1)} - v^{(3)}]. \tag{8}$$

By solving this system, we obtain that:

$$b^{(1)} = \frac{v^{(1)}}{1-\alpha^2} + \frac{\alpha}{1-\alpha^2}E[v^{(2)}] - \frac{\alpha}{1-\alpha}v^{(3)}. \tag{9}$$

The strategy of bidder 2 is symmetric:

$$b^{(2)} = \frac{v^{(2)}}{1-\alpha^2} + \frac{\alpha}{1-\alpha^2}E[v^{(1)}] - \frac{\alpha}{1-\alpha}v^{(3)}. \tag{10}$$

For the calculation of the expectation of the opponent's value, it is worth noting that in the second stage bidder knows $v^{(3)}$ and can update his expectation of opponent's value. If values are uniformly distributed on the interval [0,100], bidder 1 knows that $v^{(2)} \sim U[v^{(3)},100]$ and $E[v^{(2)}] = 0,5[100 - v^{(3)}]$. The same holds for bidder 2 whose expectation is $E[v^{(1)}] = 0,5[100 - v^{(3)}]$.

Results of the simulation

We have conducted a numerical simulation where bidders values are private and drawn from uniform distribution on the interval [0,100]. Five bidders participate in the auction and the experiment is repeated 60 times. For each auction we have determined the 3 highest values. For the two finalists who enter the second stage, we have determined the equilibrium bid and bids according to the 3 heuristic strategies. The parameter α is set at 0,5, which means that the total difference between the second and the third highest bid is distributed to two finalists. We have calculated the premium according to the equilibrium bidding strategy as the difference between the runner up's bid and the third highest equilibrium bid (the third highest bid in the first stage). We have also calculated the difference between the bids according to the 3 heuristic strategies and the equilibrium bid (overbidding).

We will first focus on the bidding strategy of the runner up. For the first heuristic strategy, runner up's bid can be either higher or smaller than the equilibrium bid, but on average this bid is higher than the equilibrium bid. For the other two heuristic strategies, bid is always higher than the equilibrium bid.

We have calculated the average overbid with respect to the equilibrium bid for each heuristic strategy of the runner up, and we have repeated the experiment 100 times. Since in each experiment, there are 60 repetitions, we have 6.000 repetitions to calculate average values and obtain stable results. The results are shown in the following table.

Table 1. Overbidding with heuristic strategies over equilibrium strategies for the runner up

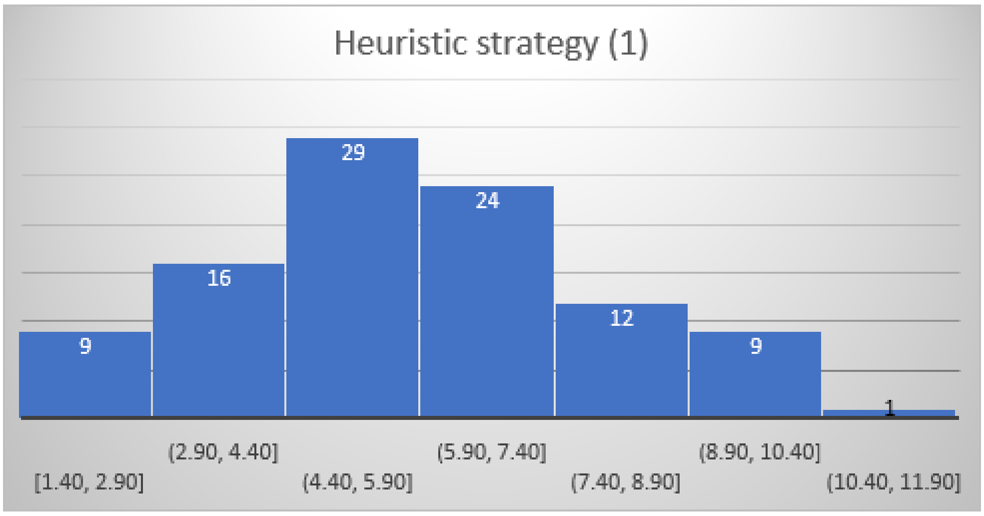
	Heuristic strategy (1)	Heuristic strategy (2)	Heuristic strategy (3)	Eq. premium
Average	**5.75**	**14.20**	**11.22**	**5.61**
Min	1.4	12.11	8.61	4.33
Max	10.43	16.4	13.86	6.94
Stdev	2.01	0.82	1.12	0.55

Source: author

Table 1 shows that the average overbidding of the runner up for the first heuristic strategy is approximately equal to the amount of the equilibrium premium. The largest overbidding is for the second heuristic strategy where the runner up has optimistic beliefs and his second order belief is that his opponent is pessimistic. The first heuristic strategy has the highest variability measured by standard deviation. It is interesting that in 44% of cases, the overbidding with the first heuristic strategy is lower than the amount of the equilibrium premium.

The histogram of the probability distribution of bids for the runner up according to the first heuristic strategy is shown in the following graph, where we can see that the probability distribution is left-skewed and it is more stretched than other two probability distributions.

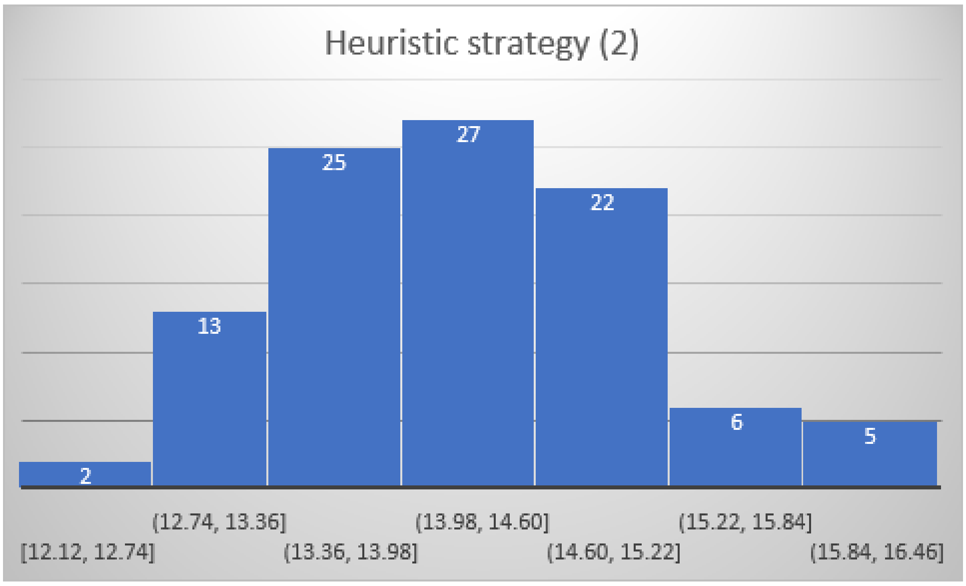
Figure 1. Histogram of bids of the runner up for heuristic strategy (1)



Source: author

The histogram for the second heuristic strategy of the runner up is shown in the following graph and this probability distribution is right-skewed, but more mass is concentrated at the central part of the probability distribution than in the previous case.

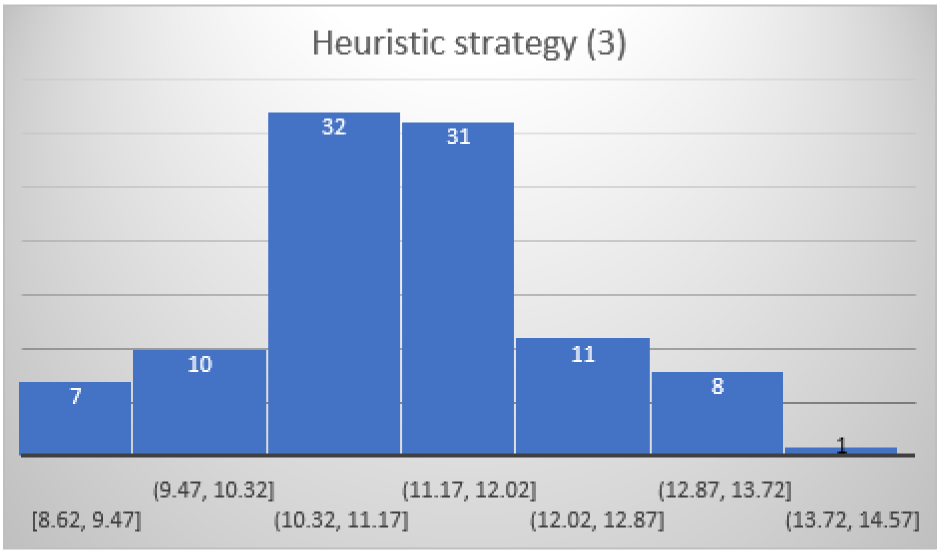
Figure 2. Histogram of bids of the runner up for heuristic strategy (2)



Source: author

The histogram for the third heuristic strategy of the runner up is shown in the following graph. If we exclude one observation at the right tail of the distribution, this distribution is almost symmetric and the mass is concentrated in the middle of the distribution.

Figure 3. Histogram of bids of the runner up for heuristic strategy (2)



Source: author

The amount of the premium with heuristic strategies is considerably larger than the amount of the equilibrium premium. After 6.000 repetitions as previously, we have determined that the amount of the premium with heuristic strategies for the runner up (the same amount of the premium goes to the winner) is at least 3 times larger than the equilibrium premium.

Table 2. Premium with heuristic and equilibrium strategies

	Eq. premium	Premium H1	Premium H2	Premium H3
Average	**5.61**	**16.97**	**21.15**	**19.681**
Min	4.33	13.2	17.9	16.25
Max	6.94	20.98	23.65	22.67
Stdev	0.55	1.84	1.27	1.43

Source: author

It is also evident that the premium is more variable with the heuristic strategies than with the equilibrium strategy.

Concerning the bidder with the highest value, his overbidding relative to the equilibrium bid is the largest for the first heuristic strategy (even though his bid on some occasions can be lower than the equilibrium bid), followed by the third and the second heuristic strategy, respectively. The winning bidder's profit in the second-price Amsterdam auction is the difference between the sum of his value and premium and the amount that he pays (the runner up's bid). With all three heuristic strategies, winner's profit is positive and always larger than the equilibrium profit.

Table 3. Winner's profit with heuristic and equilibrium strategies

	Profit for H1	Profit for H2	Profit for H3	Eq. profit
Average	**16.67**	**12.48**	**13.96**	**11.26**
Min	12.8	8.88	10.32	7.76
Max	21.15	16.35	18.04	14.93
Stdev	1.98	1.72	1.77	1.71

Source: author

The largest profit is obtained with the first heuristic strategy. This larger profit of the winner comes at the expense of the seller who obtains lower revenue, which is lower than equilibrium revenue. It is interesting to observe that seller's revenue is the same for all 3 heuristic strategies used by bidders.

Table 4. Seller's revenue with heuristic and equilibrium strategies

	Revenue H1	Revenue H2	Revenue H3	Eq. Rev
Average	**49.8**	**49.8**	**49.8**	**66.72**
Min	42.97	42.97	42.97	61.95
Max	55.93	55.93	55.93	71.47
Stdev	2.68	2.68	2.68	1.69

Source: author

This result shows that bidders can increase the premium at the expense of seller even without collusion if they use these heuristic strategies. Thus, there is a possibility for bidders to game the auction mechanism in the symmetric case. The striking result, is that this type of manipulation is possible even without the cartel agreement and the collusion would strengthen their ability to game the auction. Our results rely on the assumption that bidders bid their value in the first stage which is probably the most plausible assumption of their behavior in the first stage. They cannot bid considerably above value, since bidders with lower values would risk to win the auction in the second stage and obtain negative profit. However, Goeree and Offerman (2004) have shown that Amsterdam auction is more useful when bidders are asymmetric than when they are symmetric.

What is important to observe is that the tacit collusion between bidders in the form of using the first heuristic strategy is stable. The following table shows that there is no incentive to deviate from the tacit agreement on coordinating heuristic strategies.

Table 5. Stability of tacit collusion

		Second highest value bidder			
		H1 strategy		Equilibrium strategy	
Highest value bidder	H1 strategy	**16.67**	**16.97**	11.47	5.61
	Equilibrium strategy	14.9	3.26	11.26	5.61

Source: author

In the first outcome of the game, both finalists play heuristic strategy 1 (H1) which is the unique Nash equilibrium of the game. In the second outcome, the bidder with the highest value plays H1 and the bidder with the second highest value plays equilibrium strategy. In the fourth outcome, both players play equilibrium strategies. The third outcome is interesting when the bidder with the highest value plays equilibrium strategy, and the bidder with the second highest value plays H1. In this case, it can happen that the bidder wit the second highest value outbids the bidder with the highest value (in 21,6% of cases), but in that case obtains negative profit, while the bidder with the highest value obtains the premium.

To prevent the previously described tacit collusion, the seller can reduce the parameter α or limit the maximal amount of the premium that could be distributed to finalists.

We have also analysed the *strongly asymmetric* case when there is one strong bidder with value uniformly distributed on the interval [100,200] and 5 weak bidders with values uniformly distributed on the interval [0,100]. In the equilibrium, weak bidders bid up to 100 in the first stage, and 100 in the second stage, while the strong bidder bids his value in both stages. We have assumed that in equilibrium weak bidders in the first stage bid according to the strategy (1) (which is consistent with bidding up to 100 in equilibrium of the first stage). In the weakly asymmetric case, it is not possible to determine equilibrium strategies and we do not have a benchmark to compare heuristic strategies with equilibrium strategies.

In the strongly asymmetric case, in the second stage bidders use only the first type of heuristic strategy, since due to the structure of the game other two heuristic strategies are no longer possible. In the following table we present the amount of overbidding for the runner up, which is quite small and with the first heuristic strategy the average bid is almost equal to the equilibrium bid (the upper limit of the support of the probability distribution of weak bidders). The equilibrium premium with strongly asymmetric bidders is almost two times larger than with symmetric bidders.

The premium with the first heuristic strategy for strongly asymmetric bidders is also larger than with symmetric bidders.

Table 6. Overbidding with heuristic strategies and premia in the strongly asymmetric case

	Overbid	Eq. premium	H1 premium
average	**0.63**	**11.17**	**16.89**
Min	-10.12	9.58	13.43
Max	6.98	13.12	22.36
Stdev	3.29	0.73	1.8

Source: author

With heuristic strategy 1 the winner obtains higher profit than in equilibrium at the expense of seller who obtains lower revenue than in equilibrium.

Table 7. Profit and revenue with heuristic strategies in the strongly asymmetric case

	Profit H1	Eq. profit	Revenue H1	Eq. Revenue
average	**66.19**	**60.9**	**66.62**	**77.75**
min	55.18	48.84	61.73	74.55
max	77.27	69.81	71.73	81.23
stdev	3.95	3.75	1.99	1.33

Source: author

Therefore, in the strongly asymmetric case bidders can also game the mechanism of Amsterdam auction by increasing the premium. However, the profit from the heuristic strategy is lower than in the symmetric case, and the reduction of seller's revenue is also lower. In the symmetric case, heuristic strategies reduce seller's revenue by 25,4% compared to equilibrium revenue, and in the strongly asymmetric case heuristic strategy reduces the profit by 14,3%.

Conclusion

In this paper we have considered heuristic strategies in Amsterdam auction. We have shown that both finalists can benefit by deviating to these strategies from equilibrium strategies and that this form of tacit collusion is stable.

Our result relies on the assumption that bidders in the first stage bid their value. However, it is an important experimental question to determine the exact behavior in the first stage. If bidders bid considerably above value in the first stage (which has not been identified in the experiments so far), this would undermine the stability of tacit collusion.

This paper provides an important insight how bidders can game the auction mechanism to obtain a premium. Nevertheless, if the seller limits the maximal amount of the premium, he can reduce the incentives for such adverse behavior. Amsterdam auction is an efficient tool for deterring standard cartel agreements, and we have shown how this mechanism could be improved to prevent tacit collusion such that Amsterdam auction remains an important tool in preventing bid rigging.

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