DOI: 10.20472/IAC.2017.33.049

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DYNAMIC OPTIMIZATION UNDER UNCERTAINTY: A CASE STUDY FOR AUSTRIAN MACROECONOMIC POLICIES.

Abstract:

Results from optimum control problems with uncertain parameters are investigated in a numerical case study for Austria. Optimal budgetary policies are calculated under varying assumptions about stochastic parameters within the framework of a problem of quantitative economic policy. An intertemporal objective function is minimized subject to the constraints of a small macroeconometric model, and approximately optimal values for federal budget expenditures and revenues are determined. It is shown that the deterministic and the fully stochastic optimal policies are rather similar. If only some parameters are assumed to be stochastic, or if covariances between different parameters are not taken into account, on the other hand, optimization results can be very different from deterministic or fully stochastic optimization results.

Keywords:

optimal control; fiscal policy; stochastic control; stochastic parameters; econometric models

JEL Classification: E62, C54, C61

1 Introduction

For a long time, there has been an intensive debate in economics as to whether fiscal and/or monetary policies should be used to influence macroeconomic variables such as employment, output, the price level, and the external balance. Monetarists in particular have argued against the discretionary use of budgetary policies for stabilization purposes because of long lags and uncertainty about the effects of these policies on the economy. Even if one is ready to accept a theoretical position that implies the potential effectiveness of fiscal policy, the argument of limited knowledge about policy effects has to be taken seriously when designing actual policy measures. Some theoretical work has been done on the influence of uncertainty upon the design of macroeconomic policies; see, for instance, Brainard (1967), Henderson and Turnovsky (1972), Turnovsky (1977), Neck (1978), Johansen (1978), Hughes Hallett and Rees (1983), Mercado and Kendrick (2006). It shows that policy uncertainty can be captured in macroeconomic models by assuming that the model parameters follow some probability distribution and by deriving optimal values or time paths of policy variables in such a stochastic setting. The analytical results obtained in this literature are necessarily based on rather restrictive assumptions and do not yield conclusions which could be readily applied to actual policy problems. Therefore, numerical results are desirable for such a purpose, and should be based on econometric models estimated from actual data for the economy to which the fiscal policies are to be applied.

The present political debate about the appropriate design of budgetary policies in Austria is a case in point for this question. On the one hand, due to the "Maastricht criteria" of the Stability and Growth Pact for the European Economic and Monetary Union, it is generally recognized in Austria (as in other European countries) that the size of the federal budget deficit should be reduced. On the other hand, there are trade-offs and side-effects associated with a policy of budget consolidation. In principle, quantitative information about the effects of budgetary policies on macroeconomic targets as well as about the intertemporal trade-offs associated with such policies can be obtained from econometric models, and recommendations to policy makers can be derived from such models within an optimization framework. Due to uncertainty about the future and about the relations between economic variables, however, it is an open question as to how reliable the results of such calculations are. Policy makers will distrust such recommendations unless they are provided with additional information about the robustness of the conclusions with respect to the uncertainty of policy effects.

In this paper, we analyze some of these issues within a problem of quantitative economic policy using an optimum control approach. We determine numerically optimal budgetary policies for Austria for the 1990s by minimizing an intertemporal objective function subject to the constraints given by an econometric model. The model, called FINPOL2, is a medium-size macroeconometric model for Austria. It relates policy and exogenous variables to objective variables of Austrian economic

policies, such as the rate of unemployment, the rate of inflation, the growth rate of real GDP, the balance of current account and the budget deficit. Moreover, we postulate an objective function for Austrian policy makers over the years 1993 to 2000 which penalizes deviations of objective variables from their desired values. The exogenous variables of the model are forecast over this planning horizon using time series methods. Next, we calculate optimal stabilization policies over this time horizon using the stochastic control algorithm OPTCON. Our main interest lies in assessing the reliability of these optimal budgetary policies. In the present paper, we concentrate on the question as to how optimal policies are affected by assumptions about uncertain model parameters, taking the structure of the model and the forecasts for the exogenous variables as given.

In order to investigate the influence of the uncertainty of parameter estimates on optimal policies, we perform several optimum control experiments. First, we assume all parameters to be known for certain, i.e. we neglect the stochastic nature of the parameters and solve a deterministic control problem. Next, we take all parameters of the model to be stochastic and use the entire covariance matrix of the parameters as estimated by the 3SLS method. The results are virtually indistinguishable from those of the deterministic optimization. On the other hand, if we take only several parameters to be stochastic (i.e., consider their second moments), and in particular, if we neglect covanriances, considerable differences arise between optimal policies, depending primarily upon the amount of covariation between the different parameters of the model. This shows the importance of properly specifying the stochastic nature of the parameters in an optimum control setting in order to arrive at reliable policy recommendations. Conclusions are drawn for conducting economic policies in the future.

2 The Econometric Model FINPOL2

The FINPOL2 model is based on traditional Keynesian macroeconomic theory in the sense of conventional IS-LM/aggregate demand-aggregate supply models. It is an updated and re-specified version of our model FINPOL1, which has been used for several optimization experiments; see, e.g., Neck and Karbuz (1995). Stochastic behavioral equations for the demand side include a consumption function, an investment function, an import function and an interest-rate equation as a reducedform money market model. Prices are largely determined by aggregate demand variables. Disequilibrium in the labor market, as measured by the excess of unemployed persons over vacancies, is modelled to depend on the real GDP growth rate and the rate of inflation, embodying both an Okun's law-type relation and a rudimentary Phillips curve. The main objective variables of Austrian economic policies, such as real GDP, the labor market disequilibrium variable (related to the rate of unemployment), the rate of inflation, the balance of payments and the ratio of the federal net budget deficit to GDP, are related directly or indirectly to those fiscal policy instruments which are used as control (policy instrument) variables, namely to federal budget expenditures and revenues.

The model, which is dynamic and nonlinear, was estimated first by OLS and then by simultaneous equation estimation methods using annual data over the period 1965 to 1992 which were obtained from the Austrian Institute of Economic Research (WIFO). All real data have dimension billions of 1983 Austrian schillings (ATS). Together with ex-post simulation results the estimates and test statistics suggest that the model provides a reasonable account of the development of economic variables over the period under consideration. For our optimization experiments, we use the estimates of the parameters obtained by three-stage least squares; the software package PC TSP, Version 4.2B was used for estimating and simulating the model. Details of the results are contained in the paper by Neck and Karbuz (2017).

3 The Optimum Control Approach

In the theory of quantitative economic policy, macroeconomic policy problems are often considered as problems of optimizing an intertemporal objective function under the constraints of a dynamic system which is subject to various kinds of uncertainties. Stochastic optimum control theory has been used in several studies to determine optimal policies for econometric models; e.g., Chow (1975), (1981), Kendrick (1981), Amman (1996). Here we use the algorithm OPTCON, developed by Matulka and Neck (1992) and extended by Blueschke-Nikolaeva et al. (2012); it determines approximate solutions for stochastic optimum control problems with a quadratic objective function and a nonlinear multivariable dynamic model under additive and parameter uncertainties. The objective function is guadratic in the deviations of the state and control variables from their desired values. The dynamic system is required to be given in a state space representation. Apart from the additive error term, a constant vector of unknown (and hence stochastic) parameters enters the system equations. As input for the algorithm, the user has to supply the system function, the initial value of the state vector, a tentative path for the control variables, the expected value and the covariance matrix of the stochastic parameter vector, the covariance matrix of the additive system noise, the weight matrices of the objective function, and the desired paths for the state and control variables.

For our simulation experiments, we chose the planning horizon as 1993 to 2000. We distinguish two categories among the variables whose deviations from desired values are to be penalized. First, there are five "main" objective variables which are of direct political relevance in assessing the performance of the Austrian economy. These are the rate of inflation ($PV\%_t$), the labor market excess supply variable (UN_t) as a measure for involuntary unemployment, the rate of growth of real GDP ($YR\%_t$), the balance of current account (LBR_t), and the federal net budget deficit as a percentage of GDP ($DEF\%_t$). In all experiments, 2% p.a. is considered to be the desired rate of inflation ($PV\%_t$), 3.5% p.a. the desired real growth rate ($YR\%_t$), and the desired levels for labor market excess supply (UN_t) and the balance of current account (LBR_t) are both set to zero. Thus, we want to achieve equilibrium on the labor market and in the external relations. The choice of a positive rate of inflation rate. The desired value for

the real growth rate is higher than the average of the last few years before the optimization period but in line with the long-term growth performance of the Austrian economy after World War II. For the deficit variable, we assume that the aim is to consolidate the federal budget deficit gradually such that the desired value of DEF_{t} is reduced by 0.3 percentage points each year, from the historical value of 3.27% in 1992 down to 0.87% in 2000.

Second, we introduce a category of "minor" objective variables. These include real private consumption (CR_t), real private investment (IR_t), real imports of goods and services (MR_f) , the nominal rate of interest (R_f) , real GDP (YR_f) , real total aggregate demand (VR_t) , the domestic price level (PY_t) , the price level of public consumption (PG_t) , nominal public consumption (G_t) , and nominal public-sector net tax revenues (T_t) , as well as the policy instrument (control) variables of federal budget net expenditures (NEX_{t}) and federal budget tax receipts (BIN_{t}). We take the 1992 historical values of these "minor" objective variables (except for R_{t}) as given and postulate desired growth rates of 3.5% p.a. for the planning horizon for all real variables, desired growth rates of 2% p.a. for the price level variables, and desired growth rates of 5.5% p.a. for the nominal variables. The rate of interest R_t has a desired constant value of 7 for all periods. There are two reasons for including these "minor" objective variables: First, assigning them the above desired values shall reflect the aim of the hypothetical policy maker to stabilize the economy in the sense of achieving smooth growth paths for real and nominal variables, i.e. some kind of "balanced growth" of total demand and its components. Second, this device serves as a substitute for imposing inequality constraints on state and control variables to prevent erratic fluctuations of these variables which are not possible in reality due to various rigidities in the economic system but which cannot be adequately captured by our model.

In the weight matrix of the objective function, all off-diagonal elements are set to zero, and the main diagonal elements are given weights of 10 for the "main" objective variables and 1 for the "minor" objective variables. The state variables that are not mentioned above get weights of zero, thus being regarded as irrelevant to the hypothetical policy maker. The weight matrix is assumed to be constant over time.

The OPTCON algorithm assumes that the values of the non-controlled exogenous variables are known in advance for all time periods of the planning horizon. In addition, starting values are required for the control variables for all time periods to initialize the iterative determination of their optimal values. For a simulation over a future planning horizon, projections (forecasts) of the exogenous (controlled and non-controlled) variables are used. We use extrapolations of these variables for the years 1993 to 2000 calculated from linear stochastic time series models of the ARMA (mixed autoregressive-moving average process) type. After several trials and applying the usual diagnostic checking procedure for the time series under consideration, we decided to model federal budget expenditures (*NEX*_{*t*}) by an ARMA(2,1) process, federal budget revenues (*BIN*_{*t*}) by an ARMA(2,2) process, the import price level (*PM*_{*t*}) by an ARMA(1,1) process, real exports of goods and services (*XR*_{*t*}) by an ARMA(2,3)

process, the inventory change variable IIR_t by an AR(1) process, and money supply $(M1_t)$ by an ARMA(2,1) process.

The forecasts from these time series models imply moderate growth of the fiscal policy variables. The extrapolation implies that the federal budget deficit (*NDEF_t*) is stabilized and falls gradually from 70.3 billion ATS in 1993 to 60.5 billion ATS in 2000. The development of the foreign sector variables PM_t and XR_t is optimistic: PM_t grows only by 1% p.a. or less, and XR_t grows by 5 to 7% p.a. *IIR_t* is positive but falling. Money supply M1_t grows by 5 to 5.8% p.a. It must be stressed that these extrapolations result from the time series models and do not involve any theoretical reasoning or additional empirical information about future government policies or global developments.

4 The Deterministic and the Stochastic Optimum

As a first step, the model was simulated over the years 1993 to 2000 using the extrapolations of all (control and non-controlled) exogenous variables from the time series models as input. This amounts to a dynamic forecast of the endogenous variables of the model; no optimization is involved in this projection, which serves as a reference for comparison with the optimization runs (labeled "projection" in the figures). Next, we performed two optimization experiments as detailed in the previous section. Here again the projections of the non-controlled exogenous variables from the time series models are used as inputs, being assumed to be known for certain, but the values of the policy instruments are determined endogenously as (approximately) optimal under the assumed objective function. For a deterministic optimization run (labeled "deterministic optimum"), we assumed all parameters of the model to be known for certain. This amounts to ignoring the uncertainty of all policy effects. For a fully stochastic optimization run (labeled "fully stochastic opimum"), on the other hand, only deterministic paths for the non-controlled exogenous variables were assumed, but the estimated covariance matrices of the parameters and the additive disturbances of the model, which are obtained from the 3SLS estimation, are taken into account when calculating optimal policies. In this case, the expected value of the same quadratic objective functional is minimized.

As shown in Figures 1 to 7, the projection scenario forecasts a recession for 1993 (or actually a continuation of the recession from 1992): growth of real *GDP* (*YR%*_t) is negative, the rate of unemployment (related to UN_t) is high, the balance of trade (*LBR*_t) exhibits a deficit, the rate of inflation (*PV%*_t) is moderate, and the ratio of the federal budget deficit to GDP (*DEF%*_t) is high. Starting in 1994, however, a period of relatively high growth is projected, which is clearly above the one obtained on average in the 1980s. In particular, there is a boom in 1995 with very high real GDP growth. The labor market excess supply variable (*UN*_t) falls gradually, with only a slight rise in inflation (*PV%*_t). High surpluses are obtained for the balance of trade (*LBR*_t), particularly from 1995 on, and the deficit variable *DEF%*_t falls gradually. The optimistic forecasts for the following years are primarily due to the favorable prospects of world

market developments as expressed by the time series extrapolations for Austrian real exports and import prices.



Figure 1: Federal Budget Net Expenditures (NEXt)











Figure 4: Labor Market Excess Supply (UNt)













In spite of the already optimistic picture of the future development of the Austrian economy provided by the projected forecast, there is still some scope for optimal stabilization policy, as can be seen from the two optimization experiments mentioned above. In particular, optimal fiscal policies are more countercyclical than projected ones and imply smoother time paths of the endogenous variables of the model. The recession of 1993 is avoided by expansionary budgetary policies. Federal budget expenditures (NEX_t) are clearly higher while federal budget revenues (BIN_t) are lower than both the projected and desired levels of these variables. This results in a positive growth rate, lower unemployment, only slightly higher inflation, but distinctly higher deficits in the trade balance and the federal budget (153.9 billion ATS for the latter in the deterministic and 159.7 ATS in the fully stochastic optimum) as compared to the projection. Also in 1994 optimal stabilization policies can be characterized as expansionary, with similar (though weaker) effects on the objective variables. The values of the instrument variables in 1995 and 1996 are close to those of the projection, with slightly more expansionary tax policy. From 1997 on, optimal budgetary policies are restrictive as compared to the projection. This results in lower growth, higher unemployment and lower inflation than in the projection scenario. The surplus of trade balance starts increasing some years later than in the projection. The federal budget is fully consolidated, with a balanced budget in 1999 and even a surplus in 2000.

Comparing the deterministic and the fully stochastic optimal policies, it is remarkable that there is nearly no difference between the deterministic and the fully stochastic optimization run. Optimal values of budgetary policy variables are very close in these two experiments, with slightly more expansionary policies (higher NEX_t , lower BIN_t) in 1993, 1996, and 1998, and slightly more restrictive policies (lower NEX_t , higher BIN_t) in 1994, 1995, 1997, and 2000 in the fully stochastic optimum; in 1999, both NEX_t and BIN_t are higher in the fully stochastic optimization run. The fully stochastic optimal policies result in growth rates ($YR\%_t$) which are always a little bit closer to their

desired values, but the balance of current account always deviates slightly more from its desired value than in the deterministic optimum. Price levels and real variables (hence also nominal aggregates) show slightly lower growth over the entire planning period in the fully stochastic than in the deterministic optimum. The optimal value of the objective function is 126,080.8 in the deterministic experiment and 151,361.4 in the fully stochastic one; if we interpret the difference as the costs of uncertainty (which must be done with grat caution), they are about 20% of the deterministic minimum costs, which seems rather small.

5 Optimal Policies under Different Assumptions about Stochastic Parameters

The similarity between the optimal deterministic and fully stochastic paths of budgetary policies seems to imply that optimal policies are reliable even when neglecting the parameter uncertainty. This is somewhat astonishing because previous theoretical and numerical studies using simple macroeconomic models have shown that uncertainty generally matters in the design of optimal stabilization policies. Therefore we would like to know whether our result also holds true when only some parameters of the model are treated as stochastic, with the other ones remaining deterministic. In Neck and Karbuz (1995), for instance, we showed that optimal policies for past periods may have been rather different, depending on which parameters were regarded as stochastic. In a similar way, we now introduce different assumptions about parameter uncertainties into our FINPOL2 model in order to assess the influence of various kinds of uncertainty on the design of future optimal budgetary policies. In particular, we want to investigate the effects of making several key parameters which determine fiscal and monetary policy multipliers uncertain. Also these experiments start from the specification of the objective function assumed for the fully stochastic optimum and described in Section 3 above.

As our FINPOL2 model was estimated by 3SLS, we have an estimate of the entire parameter covariance matrix. To concentrate on the effects of uncertainty of some key parameters, we neglect the variance and covariance estimates of the other parameters. However, it turns out that taking covariances between different parameters into account or not may be crucial for optimal policies. Therefore, in the following experiment, we calculated two versions of optimal budgetary policies. For the first version (labeled "stochastic policy multipliers, no covariance" in the figures), only the variances of the parameters regarded as stochastic were taken into account and all covariances were neglected. In this case, the parameter covariance matrix is diagonal, with non-zero elements in the main diagonal only for the stochastic parameters. The second version (labeled "stochastic policy multipliers, covariance") also takes into account the covariances between the stochastic parameters and all other parameters (which still have zero variances) are given non-zero values. In this way, we want to study the effects of correlations between parameter estimates on the design of optimal policies. The estimates for parameter variances and covariances were always taken from the estimated parameter covariance matrix obtained from the

3SLS estimation. The results can be compared either to the deterministic optimum or to the fully stochastic optimum (which is very similar to the deterministic one anyway).

Several experiments along these lines were conducted; see Neck and Karbuz (2017) for details. The most interesting one from the point of view of economic policy design arethose where we assume that only the coefficients relating to monetary and fiscal policy (the policy multipliers) are uncertain. To do so, we take as stochastic parameters the marginal propensity to consume, the coefficient of the real rate of interest in the investment equation and the coefficients of real money supply and real GDP in the interest rate equation. In the version "stochastic policy multipliers, no covariance" of this experiment, optimal NEX_t and optimal BIN_t values are higher than in the deterministic optimum, resulting in a larger public sector. The results of version "stochastic policy multipliers, covariance" are completely different from those in version "stochastic policy multipliers, no covariance". Except for the last year of the planning period, budgetary policies act in an expansionary way, with NEX_t and especially BIN_t (except for 2000) set at lower values than in the deterministic optimum.

The extent to which optimal budgetary policies and their performance depend upon the number of correlations between the stochastic parameters of the model can also be seen from the results of another experiment (labeled "fully stochastic optimum"). Here we assume that all estimated parameters of the model are stochastic but neglect their covariances, i.e., we only take into account the estimated variances of the 37 parameters. The results shall be compared to the "fully stochastic optimum" with the covariances taken into consideration. Here the optimal budgetary policies are dramatically different from both the deterministic ones and the fully stochastic ones with parameter covariances taken into account. They are even more restrictive than those in version ""stochastic policy multipliers, covariance" (the experiment with stochastic fiscal policy multipliers). High federal budget tax revenues and low federal budget expenditures combine to create a budget surplus for all but one year. A severe recession is created in 1993, with real GDP falling by more than 12% and labor market excess supply jumping up to more than 9%. Intertemporal trade-offs are exploited to some extent, because UN_f comes down to 1.5% until 2000, inflation remains low, and the balance of current account has a big surplus in every year. From this, we may conclude that neglecting parameter covariances in stochastic optimization problems results in optimal policies which are heavily biased compared to the fully stochastic optimum with parameter covariances taken into account. This result concurs with the observations by Amman and Kendrick (1999), although in their framework (which includes a more sophisticated treatment of uncertainty, involving also learning about parameters) also the neglect of variances appears to be highly misleading.

6 Concluding Remarks

In this paper, we have used a medium-size macroeconometric model of the Austrian economy to calculate optimal budgetary policies for the years 1993 to 2000 for a

given objective function. In particular, we have investigated the effects on optimal budgetary policies of making some parameters uncertain that are important for the transmission of policy effects between objective variables. The results show that optimal budgetary policies for the particular model employed are very sensitive with respect to stochastics affecting fiscal policy multipliers and especially to correlations of these parameters with other ones. On the other hand, if the full covariance matrix of the parameters is taken into account, optimal policies are similar to those obtained in the deterministic case. We interpret this result as showing that neglecting correlations between model parameters leads to policy recommendations which ought not to be presented to policy makers as they are seriously flawed. This result is important for practical policy advisers because often econometric models are estimated by OLS and hence no estimate for the entire parameter covariance matrix is available. In this case, results based on deterministic optimization may even be preferable to those based on incomplete stochastics.

From a more general perspective, if we compare the results of the deterministic or the fully stochastic optimization run to a simulation with extrapolations of policy instruments used as inputs, optimal policies turn out to be more countercyclical and to dampen the amplitude of business cycle fluctuations. If this is in fact a goal of economic policy making, using an optimum control approach within a framework of quantitative economic policy could be recommended to political decision makers and their advisers as an instrument to generate insights into possibilities for improving policy making. In particular, discretionary countercyclical budgetary policies can lead to improvements with respect to macroeconomic target variables. In order to obtain reliable recommendations for such policies, parameter uncertainty should be either fully taken into account or, if this is not feasible, neglected completely. Anyway, still more research is needed, especially with more elaborate econometric models, before policy proposals can be derived which can be implemented for actual political decisions.

Acknowledgement

Financial support from the "Jubiläumsfonds der Oesterreichischen Nationalbank" (project no. 5126) is gratefully acknowledged. Karbuz acknowledges support from the Institute for Advanced Studies, Vienna.

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