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# INCREASING RETENTION IN MATHEMATICS COURSES: THE ROLE OF SELF-CONFIDENCE IN MATHEMATICS ON ACADEMIC PERFORMANCE 


#### Abstract

: Underachievement rates in mathematics for the United States have been alarming for a long time. While the reasons have been studied at length, a large area pays close attention to self-confidence as predictor of academic performance. Most research on this area however, is based on high school students. This study extends this line of work by assessing self-confidence and its effect on academic performance among college students. Using quantile regression we show that self-confidence positively impacts class performance for the middle and bottom quantiles, but not the top 75th percent. These results imply that simple and costless confidence boosting exercises conducted in the classroom may have a positive impact on at risk students, and consequently retention. The results appear to be generalizable, rather than localized to summer school students.


## Keywords:

Retention, self-confidence, mathematics, Fennema-Sherman, academic performance

## Introduction

Over the past decades the United States has faced alarming rates of underachievement in mathematics among students of all ages, which at best have remained constant over the recent years. According to the National Assessment of Educational Progress (NAEP) average performance scores in mathematics for $12^{\text {th }}$ graders were below the proficient level in 2013, and they were no different from 2009 (NAEP, 2013). One of the major goals of the No Child Left Behind Act of 2001 was to ensure that, at a minimum, all students in the US performed at a mandated proficiency level in reading and mathematics. The most recent results from NAEP show that we are far from meeting this goal.

These rates of underachievement are not only high in absolute terms, but high relative to other developed countries (OECD, 2013; McKinsey \& Company, 2009; Borba, 2005), and gender, racial/ethnic and socio-economic differences are heavily pronounced (NAEP, 2013; Tang, 2010; McKinsey \& Company, 2009; Brandell, Leder \& Nystrom, 2007; Gallagher \& Kaufman, 2005; Gurian, Henley \& Trueman, 2001). In particular, NAEP estimates show that achievement gaps across gender and racial/ethnic twelfth-graders have not changed much since the early 90 's whereby males outperform females, and white students outperform ethnic minorities (NAEP, 2013). These under achievements and gaps in performance are also echoed in the college years, and they are in part due to lack of academic preparation as well as other social factors (e.g. Balduf, 2009; Haycock \& Huang, 2001).

As argued in the media, the end result is a graduating workforce lacking quantitative reasoning and problem-solving ability, skills of high interest to employers, and essential for the modern workplace (Belkin, 2015; Crotty, 2014). Current advances in technology offer feasible opportunities to reform the 'traditional' curriculum with modern, scalable, and equitable methods that can potentially address concerns about classroom/school effects, which contribute to student underachievement, particularly among minority students (e.g. Gutierrez, 1996; 2000; Boaler, 2002a, 2002b, 2015). Nonetheless, while the subject of mathematics is emotionless - an axiomatic process of deductive reasoning - the individuals learning mathematics are not. As such, we cannot ignore the emotional or psychological component of learning mathematics and its likely effect on student performance (Mattarella-Micke, Mateo, Kozak, Foster \& Beilock, 2011; Brown, Brown \& Bibby, 2008; Yenilmez, Girginer, \& Uzun, 2007; Ashcraft \& Krause, 2007; Rubin, Bukowski \& Parker, 2006; Gumora \& Arsenio, 2002).

In fact, since the 20th Century, researchers have paid close attention to psychological factors such as individual perceptions of ability (i.e. self-confidence) as predictor of academic performance (e.g. Broadbooks, 1981; Sherman, 1983; Sherman \& Fennema, 1977; Fennema \& Sherman, 1976) and have concluded that self-confidence can explain differences in performance just as much or even more than actual intellectual ability in the
subject (Kaufman, 2011). Most research on this area however, is based on high school students. This study seeks to extend this line of work by assessing self-confidence and its effect on academic performance among college students from a large urban minority serving university in the Northeastern part of the United States.

## The Present Study

The main objective of this study was to examine the relationship between self-confidence in mathematics ability and performance in mathematics courses among undergraduate students. Study participants $(\mathrm{N}=204)$ were recruited from a public four-year college in the Northeast (USA). More specifically, we used an online recruitment tool to recruit undergraduate students from various disciplines who were enrolled in mathematics course(s). Participants were enrolled in courses that ranged from basic to advanced, such as basic statistics, calculus I, calculus II and vector calculus. All questionnaires were completed in a research lab. Completion time ranged from 25 to 30 minutes, and there were no more than four respondents completing the questionnaires at a time. This study and all associated procedures were approved by the Institutional Review Board of the university, and written informed consent was obtained from all participants prior to enrollment. All respondents received course credit for their involvement in the study.

## Measures

Self-Confidence in Mathematics: Individual self-confidence in mathematics ability was assessed using the subscale for confidence from the modified Fennema-Sherman Mathematics Attitude Scale (Fennema \& Sherman, 1976). Items are measured on a 5 point Likert Scale ( $1=$ Strongly Agree ... 5 = Strongly Disagree). Sample items for this subscale include "I am sure I can learn math" or "Math is hard for me". This subscale had high internal reliability according to Cronbach's alpha ( $\alpha=0.92$ ).

Performance: Student performance was gauged using final average scores for each student. Average scores were obtained from faculty directly at the end of the summer term, and verified through the college's central grade portal for accuracy.

Demographics: Participants also provided data on gender, age, education status (freshman, sophomore, junior, senior or graduate), major, and number of mathematics classes taken prior to the current course.

## Results

## Sample Characteristics

All descriptive statistics appear in table 1. Respondents had a mean age of 22.31 ( $S D=4.20$ ) and were between the ages of 18 and 38 years old, with approximately $9 \%$ of the sample being 30 years or older ( $8.5 \%, \mathrm{n}=18$ ). The sample was characterized by $57.9 \%(n=117)$ males and $42.1 \%(n=85)$ female. Respondents represented a variety of
majors within the areas of Social Science (14.34\%, n = 29), Science (14.9\%, $\mathrm{n}=30$ ), Engineering (56.9\%, $\mathrm{n}=115$ ), Education ( $8.4 \%, \mathrm{n}=17$ ) and a few undecided majors ( $5.5 \%$, $\mathrm{n}=11$ ). All typical education statuses were well represented in the sample including freshmen ( $9.5 \%, \mathrm{n}=20$ ), sophomores ( $31.1 \%$, $\mathrm{n}=66$ ), juniors ( $35.4 \%, \mathrm{n}=75$ ), seniors ( $19.3 \%, \mathrm{n}=40$ ) and other ( $4.7 \%, \mathrm{n}=10$ ) representing non-degree students. The latter we believe is mostly represented by students from other campuses or schools taking summer courses at this institution.

Gender and choice of major were significantly dependent with a large effect size $\left(\chi(4)^{2}=25.95, p=0.00, V=0.36\right)$, whereby females were more likely than males to major in education ( $z=3.54, p=0.00$ ), science ( $z=2.88, p=0.00$ ) and social science $(z=2.84, p=0.00)$. Males were more likely than women to major in engineering $(z=6.43, p=0.00)$. Neither gender was more or less likely to be undecided about their majors $(z=-0.09, p=0.47)$. The modal response for number of prior math classes taken was between 2 and $4(60.9 \%, n=123)$, with $24.8 \%(n=50)$ of the sample having taken 5 or more, and $14.4 \%(n=29)$ having taken less than 2 .

Table 1: Sample Characteristics ( $\mathrm{N}=204$ )
\% or Mean (SD)

## Gender

## Male <br> Female <br> Education Status

57.90\%
42.10\%

Freshman 9.40\%
Sophomore 31.20\%
Junior
35.60\%

Senior
18.80\%

Other $5.00 \%$

## Major

| Education | $8.40 \%$ |
| :--- | :--- |
| Engineering | $56.90 \%$ |
| Science | $14.90 \%$ |
| Social Science | $14.40 \%$ |
| Undeclared | $5.50 \%$ |
| Age | $22.3(4.2)$ |
| Prior Mathematics Classes Taken |  |
| 0 to 1 | $14.40 \%$ |
| 2 to 4 | $60.90 \%$ |
| 5 or More | $24.80 \%$ |
| Confidence and Usefulness | $45.89(9.04)$ |
| Self Confidence in Mathematics |  |

Source: Own data

## Construct Validity of the Self-Confidence Scale

In addition to computing Cronbach's alpha ( $\alpha=0.92$ ), a single Confirmatory Factor Analysis (CFA) with maximum likelihood estimation using the sample covariance matrix as input was used to assess the psychometric properties of the construct. The results, presented in table 2, indicate a good fit to the data $\left(\chi(40)^{2}=40.82, p=0.43\right)$. All factor loadings were statistically significant $(p<0.05)$. The root mean square error of approximation (RMSEA), the comparative fit index (CFI), the Tucker-Lewis index (TLI) and the standardized root mean square residual (SRMR) also indicate a good fit ( $R M S E A=0.010 ; C F I=0.999 ; T L I=0.999 ; S R M R=0.03$ ). General rules of thumb guidelines representing a good fitting model are CFI $\geq 0.95$, TLI $\geq 0.95$, SRMR $\leq 0.09$ and RMSEA $\leq 0.06$ (Hooper, Coughlan \& Mullen, 2008; Hu \& Bentler, 1999). We also analized composite reliability, as well as convergent validity by computing the construct reliability (CR) and the average variance extracted, and verifying they meet the 0.7 \& 0.5
benchmarks respectively (Bagozzi \& Yi, 1988). As shown the self-confidence scale shows composite reliability and convergent validity with a construct reliability ( $C R=0.91$ ), and an average variance extracted $(\mathrm{AVE}=0.50)$ that meet the required criteria.

Table 2: Standardized Factor Loadings and Reliability Statistics

| Item Description | Standardized <br> Factor loadings |
| :--- | :--- |
| I am sure that I can learn math. | $.617^{* * *}$ |
| I don't think I could do advanced math. | $.617^{* * *}$ |
| Math is hard for me. | $.731^{* * *}$ |
| I am sure of myself when I do math. | $.609^{* * *}$ |
| I'm not the type to do well in math. | $.805^{* * *}$ |
| Math has been my worst subject. | $.765^{* * *}$ |
| I think I could handle more difficult math. | $.634^{* * *}$ |
| Most subjects I can handle OK, but I just can't do a |  |
| good job with math. | $.776^{* * *}$ |
| I can get good grades in math. | $.556^{* * *}$ |
| I know I can do well in math. | $.695^{* * *}$ |
| I am sure I could do advanced work in math. | $.665^{* * *}$ |
| I'm no good in math. | $.782^{* * *}$ |

Chi-Square $=40.82, \mathrm{df}=40, \mathrm{p}=0.434 ; \mathrm{RMSEA}=0.01 ; \mathrm{CFI}=0.99 ; \mathrm{TLI}=0.99 ;$
$S R M R=0.03 ; C D=0.92 ; A V E=0.50 ; C R=0.91$

Source: Own Data.

## Predictors of Academic Performance

On average students had high performance $(M=78.42$, $S D=18.07)$. However, performance was negatively skewked (skewness $=-1.95$ ), meaning that the majority of the students performed above average ( $61 \%, \mathrm{n}=124$ ). No differences in performance were detected across gender $(F(1,200)=0.05, p=0.83)$, or major $(F(4,197)=0.52, p=0.72)$. We did notice differences in performance across number of math classes previously taken with a small effect size $\left(F(2,199)=3.70, p=0.0 .03, \eta^{2}=0.03\right)$. Specifically, students who prior to this class had taken between 2 and 4 classes had higher performance on average ( $M=80.62, S D=15.90$ ) than those who had taken from 1 to none ( $M=70.73, S D=22.90$ ) and this difference was statistically significant at the $95 \%$ confidence level ( $p=0.02$ ) according to pairwise comparisons with the Bonferroni adjustment. Due to the natural skewness of the performance data, we confirmed the tests above with Kruskal-Wallis tests of equality of populations rank tests. The results were unchanged. Finally, the correlation between confidence and performance was positive and statistically significant ( $\rho=0.225, p=0.00$ ).

Due to the negative skew in academic performance, estimation methods that assess the average relation between performance and other variables may be biased, as they are very sensitive to outliers, and have strict assumptions about the distribution of the error terms. We therefore used non-parametric regression methods to assess predictors of academic performance. Specifically, we performed quantile regressions (Koenker and Bassett, 1978) to predict the effect of confidence, controlling for other independent variables, on academic performance. Unlike least-squares regression, which predicts mean performance, quantile regression predicts specified percentiles of the dependent variable by minimizing the sum of absolute residuals rather than the sum of squared residuals, and is a valid alternative over regression methods that incorporate violations of model assumptions (Gould, W. \& Rogers, W.H, 1994). Moreover, simultaneous quantile regression allows for the estimation of multiple percentiles, which in turn allows one to test if the effect of any given variable is the same at different percentiles. All standard errors were computed via 5,000 bootstrap replications. The results appear in table 3 .

As shown, the effect of self-confidence in mathematics ability ranges in size between the $25^{\text {th }}, 50^{\text {th }}$ and $75^{\text {th }}$ percentiles. In particular, the effect of confidence for the $75^{\text {th }}$ percentile is not different from zero statistically. Also, although the effect of confidence appears stronger for the $25^{\text {th }}$ percentile, than for the $50^{\text {th }}$, the difference between these two effects is not statistically significant $(F(1,196)=0.30, p=0.583)$. Figure 1 presents these effects graphically.

Interestingly, an indicator for number of mathematics courses taken prior is only significant for the $75^{\text {th }}$ percentile. This implies that for students at the top of the
distribution, experience taking mathematics matters more than self-confidence. The inverse is true for students who are at the median or below.

Table 3: Quantile regression of academic performance ( $\mathrm{N}=204$ )

|  | 25 th Percentile |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | b | SE | $95 \% \mathrm{CI}$ |  |
| Male | -0.13 | 4.08 | -8.19 | 7.92 |
| Age | $-0.75^{*}$ | 0.3 | -1.34 | -0.17 |
| Education Major | 1.72 | 6.74 | -11.56 | 15.01 |
| Engineering Major | -7.57 | 4.7 | -16.83 | 1.69 |
| Science Major | -1.46 | 5.49 | -12.29 | 9.37 |
| Confidence (Factor Scores from CFA) | $11.61^{* *}$ | 4.42 | 2.9 | 20.32 |
| Took 2 or more math classes prior | 3.6 | 4.64 | -5.56 | 12.76 |
| Constant | $92.65^{* * *}$ | 7.83 | 77.2 | 108.1 |
| R-Squared (pseudo) | 0.08 |  |  |  |
|  | 50 th Percentile |  |  |  |
| Education Major | b | SE | $95 \% \mathrm{CI}$ |  |
| Engineering Major | $-9.00^{* *}$ | 2.74 | -14.41 | -3.6 |
| Science Major | -4.62 | 3.44 | -11.4 | 2.17 |
| Confidence (Factor Scores from CFA) | $9.19^{* *}$ | 3.12 | 3.05 | 15.34 |
|  |  |  |  |  |


| Took 2 or more math classes prior | 5.86 | 3.03 | -0.11 | 11.83 |
| :--- | :--- | :--- | :--- | :--- |
| Constant | $97.59^{* * *}$ | 7.1 | 83.6 | 111.58 |
| R-Squared (pseudo) | 0.09 |  |  |  |
|  | 75 th Percentile |  |  |  |
| b | SE | $95 \% \mathrm{Cl}$ |  |  |
| Male | 0.93 | 1.82 | -2.65 | 4.51 |
| Age | -0.34 | 0.25 | -0.83 | 0.15 |
| Education Major | -3.43 | 3.29 | -9.92 | 3.06 |
| Engineering Major | $-7.03^{* *}$ | 2.05 | -11.07 | -3 |
| Science Major | -2.93 | 2.41 | -7.7 | 1.83 |
| Confidence (Factor Scores from CFA) | 4.2 | 2.55 | -0.82 | 9.23 |
| Took 2 or more math classes prior | $6.37^{*}$ | 2.54 | 1.35 | 11.38 |
| Constant | $95.84^{* * *}$ | 5.44 | 85.1 | 106.57 |
| R-Squared (pseudo) | 0.10 |  |  |  |

Source: Own Data; Dependent Variable = Final average scores by student, $b=$ Point estimate, $S E=$ Bootstrapped standard error with 5,000 replications, Cl = confidence interval. "p<. 05 two-tailed, " $p<.01$ two-tailed, "** $p<.0001$ twotailed. The reference groups correspond to females, social science majors who have taken less than 2 math courses prior to course taken while participating in the study.

Figure 1: Estimated effect of confidence on academic performance (results by quantile)


Source: Own Data

## Comparing summer and fall Students

With the understanding that some results may only generalize to the population of students who opt to take mathematics classes over the summer, we conducted the study again over the fall semester. While 185 students participated, numeric average grades were only available for 100 students. To increase reliability of any comparison across semesters, we are repeating the study over subsequent terms. However, preliminary findings show that no differences are likely. In particular, quantile regression estimates for these 100 students show that confidence positively impacts grades for the $25^{\text {th }}$ and $50^{\text {th }}$ percentile ( $b=48.29, p=0.03 ; b=34.20, p=0.01$ ) respectively, but not for the $75^{\text {th }}$ percentile $(b=16.55, p=0.07)$. These results are similar to the results found with summer students.

## Conclusion

This paper addressed the relation between confidence in mathematics, and performance in mathematics courses among undergraduate students. The results show that selfconfidence in mathematics affects academic performance, but the effects are not constant across quantiles. In particular, the impact of self-confidence on academic
performance is strongest for the $25^{\text {th }}$ and the $50^{\text {th }}$ percentiles, and not significant for top students. For this latter group, the number of mathematics courses taken is what is relevant. These results imply that simple and costless confidence boosting exercises conducted in the classroom may have a positive impact on at risk students, and consequently retention.

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