Abstract:
This article examines the ability of the output gap to forecast inflation, while analyzing Czech, Slovak, Hungarian and Polish economies. Output gaps are estimated using the Hodrick-Prescott filter and the production function. A simple Phillips curve model is chosen to explore their forecasting capabilities. Forecasting errors are then compared with an AR(1) process. In general, it holds true for all economies in question that integration of the output gap into models enhances their ability to forecast inflation.

Keywords: Inflation Forecasting, Output Gap, Potential Product, Phillips Curve, HP Filter, Production Function

JEL Classification: C53, E32, E37

1 Introduction
Central banks often use potential product and output gap for determining the current state of an economy and the subsequent monetary policy decisions. Potential product is often defined as such level of real output that does not cause the emergence of internal imbalances, especially rising inflation. Output gap is then typically calculated as a ratio of real and potential output. However, one of the limitations of these variables is that they are unobserved by nature. Therefore, there exists an uncertainty regarding their exact value; to overcome this, they are often estimated using multiple methods. These can be in general divided into four main groups: univariate methods, structural models, multivariate models and a relatively scarcely used direct measurement. Despite these limitations, output gaps have a theoretical ability to forecast inflation. It should hold true that when real output surpasses potential output (an economy is in the area of a positive output gap), inflation should be on the rise, and vice versa.

The aim of this paper is to find out whether including output gaps into models can improve their ability to forecast inflation; we have chosen four sample countries: the Czech Republic, Slovakia, Hungary and Poland. To find out whether this proposition holds true, we will compare the Phillips curve relationship with AR(1) process. We will calculate output gaps by two mainstream methods: the Hodrick-Prescott (HP) filter and the production function. The structure of this study is as follows: after introduction, the second chapter deals with the Phillips curve and its uses in modelling techniques. Thereafter, we will compute output gaps for the four countries in question, and examine revisions of output gaps stemming from the methodology itself. The fourth chapter will then answer whether these output gap estimates help to forecast inflation in the four selected countries, using different model
setups. The subsequent chapter would try to hint at possible improvements in models in order to obtain more accurate methods for inflation forecasting; the sixth section concludes the paper.

2 Phillips curve

The relationship between the change in nominal wages and the unemployment rate, known as the Phillips curve (Phillips, A. W. H.m 1958), has found a relatively quick use by policy makers. According to the original Phillips' theory, a low unemployment implies pressures to labour demand and therefore to nominal wage growth. After several years, the mainstream theory switched from change in nominal wages to inflation, i.e. in policy making decisions, there should be a trade-off between unemployment and inflation. This version of Phillips curve was nevertheless subject to much debate. For example, Phelps (1967), Friedman (1968), Lucas (1976), Fisher (1977) and Taylor (1980) argued, that there is no stable relationship between present unemployment and future inflation, if the structure of economy changes. Another modification of the original Phillips curve is the NAIRU\textsuperscript{i} Phillips curve, stating that there indeed exists a certain relationship between unemployment and future inflation, that allows for inflation forecasts to a certain extent.

Phillips curve was then subject to research focusing on its assumptions of symmetry and linearity. Razzak (1995) examined these properties in the case of New Zealand and found out that they are not realistic. Again, Laxton, Rose and Tetlow (1993) reached asymmetry and non-linearity of the Phillips curve in the case of the Canadian economy. Similar negative results were achieved also when examining other countries. Laxton (1994) found out that in G7 economies, there was much greater increase in output increasing inflation than decrease in output diminishing inflation for the years 1967-1991. Turner (1995) came up with a statement that during 1960-1993, the inflation effect of positive output gap was much greater than deflation effect of negative output gap, again for G7 countries.

As hinted in the previous paragraphs, several studies argued that it is better to replace unemployment gap by the output gap, which should also provide for better inflation forecasting. According to these models, it should hold true that when an economy rests above its potential, or accordingly, is in a positive output gap territory, there should be upward pressures on inflation, and vice versa. Outcomes of studies examining this relationship are quite dissimilar. Coe a McDermott (1997) confirmed this relationship for selected Asian countries and Australia using output gap calculated on the basis of non-parametrical regression. Claus (2000) examined inflation forecasting abilities of the output gap using three selected methods in the case of New Zealand and found out that output gap is a plausible explanatory indicator for inflation. Similar outcome was reached by Dwyer et al. (2010) for the British economy. Nevertheless, Atkenson and Ohanian (2001) argued that NAIRU Phillips curve may lead to false conclusions regarding future inflation in case of the US economy. Stock and Watson (2008) went through multiple studies in this area and stated that the majority of papers related to the ability of the output gap to forecast inflation using the Phillips curve failed for the specific cases of the USA and Canada.

We are not aware of any studies regarding the ability of output gaps to forecast inflation in the four countries we examine, i.e. the Czech Republic, Slovakia, Hungary and Poland. On the other hand, there is an abundance of studies regarding the estimation of output gaps themselves in these countries. Hájek and Bezděk (2001) estimated the potential product and output gap for the Czech Republic using the HP

There are also several studies dealing with potential product of the Slovak economy, namely Galabová et al. (2005) or Zimková and Bachorovský (2007), who estimated the production function by the same methodologies as we do in this study - the HP filter and production function. Konuki (2008) calculated the potential product using the HP filter, production function and multivariate Kalman filter, whereas the third method was deemed to be the most appropriate for the Slovak economy. The output gap for Poland was estimated by Gradzewicz and Kolasa (2004) using the vector error correction model (VECM); Epstein and Macchiarelli (2010) used the production function. Darvas and Vadas (2003) estimated the potential product and output gap for Hungary using univariate methods. This approach was complemented by structural and multivariate methods by Benk et al. (2005). Neo-keynesian Phillips curve was analysed in the case of Hungary by Menyhért (2008); inflation targeting for Hungary was examined by Siklos and Abel (2002). Output gap estimates for the Czech and Hungarian economies using the DSGE model were undertaken by Vetlov et al. (2011).

3 Selected methods of production function estimates

For the actual analysis of the ability of the output gap to forecast inflation, we use two methods of calculating the potential product: the Hodrick-Prescott (HP) filter and the production function. These methods can be today considered as a mainstream and are widely used by many both national and international institutions, despite their limitations that will be discussed further on.

Hodrick-Prescott filter

The Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) is one of the commonly used methods to obtain potential output. Its main advantage are low data demands (only real GDP time series), and a relatively easy application. This method decomposes the real output \( y_t \) into its trend part \( y_t^* \) and the cyclical part \( c_t \): \[
y_t = y_t^* + c_t \quad \text{for } t = 1, \ldots, T
\] (1)

Under the assumption that the trend part is continuous, we can then approach the actual estimation using this formula:

\[
\min \left\{ \left( y_t^* \right)^2 \sum_{j=1}^{T} \left( y_t - y_t^* \right)^2 + 2 \sum_{t=2}^{T-1} \left[ \left( y_t^* - y_{t+1}^* \right) - \left( y_t - y_{t+1} \right) \right] \right\}
\] (2)

The equation (1) is encompassed in the left part of equation (2) and expresses the sum of squared differences between the actual and potential product, i.e. the output gap. The right part of the equation
presents squared differences in potential product growth. The trend part $v_t^*$ is then estimated minimizing the sum of squared cyclical part of real output, penalised for second differences in the trend part. The $\lambda$ parameter can be changed in order to set the degree of penalization of the variability in the trend part. It needs to be a positive number, and the higher its value, the smoother is the trend part. In this study, all data for all economies will be estimated using the standard value $\lambda=1600$.

Another commonly cited feature of the HP filter is the end-point bias, that is particularly prominent in cases where a time series begins and ends at different parts of the business cycle. Output gap estimates at ends of such time series could be then distorted. In our case, we would refrain from using techniques for alleviating this problem (with the exception of trend TFP) - applying the filter also on forecast values being one of them - which could possibly hamper the interpretation of results, as individual approaches in every country/variable would have to be used.

**Production function**

The main advantage of the HP filter is the relative simplicity of calculation, the resulting time series is however not easy to interpret as we are not able to determine driving forces of potential product growth. This is partially overcome by the production function that uses more relevant economic variables for the description of potential product and estimates should therefore be more precise. There exists many types and setups of the production function. In this study, we will use the mainstream Cobb-Douglas function, with Hicks-neutral technological process. Real GDP at time $t$ ($Y_t$) can be then decomposed into three parts: the total factor productivity $A_t$, capital stock $K_t$ and employment $L_t$. In line with most other studies, we will use constant returns to scale. In order to keep the parsimony of the model and to avoid possible methodological and data problems, we will also use constant capital and labour weights set at $\alpha = 0.35$, in line e.g. with d'Auria et al. (2010).

$$Y_t = A_t \cdot K_t^\alpha \cdot L_t^{1-\alpha}$$

For calculating the potential product, it is necessary to determine potential levels of all three components. For the capital stock, we will adhere to the standard practice, i.e. we will make equal the actual and potential capital stock. This will be calculated using the perpetual inventory method, i.e. the current capital stock is the sum of current fixed investments and past capital stock adjusted for depreciation. The annual depreciation rate is set at 0.05, in line with e.g. Mourre (2009).

It is convenient to further decompose potential employment into three items, as seen in equation (4), multiplying working age population (15-64), potential participation rate - which we set equal to trend participation rate using the HP filter - and the NAIRU. This variable will be estimated using the methodology in Elmeskov (1993), using the relationship between nominal wage growth and change in unemployment rate.

$$L_t^* = pop_t^{15-64} \cdot part_t^* \cdot (1 - NAIRU_t)$$

The last step is to determine the potential total factor productivity. In order to keep the model as simple as possible, we decided to again make equal the potential and trend TFP using the HP smoothing. As
the end-point bias could be detrimental in this case, first, we make a two year ahead TFP forecast using linear trend and the HP filter is then applied on the whole time series.

**Figure 1: Comparison of output gaps as calculated by the HP filter and the production function (as a % of potential output)**

![Graphs showing output gaps for Czech Republic, Slovakia, Poland, and Hungary](image)

Source: Eurostat, own calculations

Figure 1 presents output gaps for all four economies. It is clear that results obtained from both methodologies differ only slightly, which is caused by the use of the HP filter at various stages of calculation of the production function approach. Especially in the case of the Czech and Slovak economies, there is a more pronounced production function-based output gap due to large slump in fixed investments and rise in cyclical unemployment.

It is necessary to point out that output gap results in Figure 1 are calculated using all available data until Q4/2012. However, output gap level at a given point of time is not final: the analysed time series as a whole changes with the progress of time and new data. The resulting output gap revisions can be quite significant. Figure 2 (lhs) shows output gap at Q1/2005 as calculated using the production function, using the data up to Q1/2005 and its gradual revisions until Q4/2012, using additional
quarters. We can see that the difference between the "actual" and "final" value can exceed 3 pps. and output gaps may even have opposite signs, see the case of the Czech Republic. When looking at the output gap in Q1/2007 (Figure 2, rhs), we can see that first calculations underestimated overheating of Czech and Hungarian economies. These revisions are then important when determining the power of output gaps to forecast inflation, as will be shown later on.

Figure 2: Subsequent output gap revisions in Q1/2005 (lhs) and Q1/2007 (rhs)

Source: Eurostat, own calculations

4 Comparison of inflation forecasting models

For the analysis of the ability of output gaps to forecast inflation\(^iv\) we applied the commonly used linear model presented in Orphanides and van Norden (2004), Hjelm and Jönsson (2010) or Clausen and Clausen (2010), where (y-o-y) inflation depends on inflation in the past period (in our setup the past quarter) and present output gap \(og\).

\[
\pi_{t+h} = \beta_1 + \beta_2 \pi_{t-1} + \beta_3 og_t + \epsilon_t
\]

We would be interested to see whether \(h\)-step-ahead inflation forecast on the basis of this model beats forecast of a simple autoregressive model, without the output gap (see equation 6). In this study, we will examine the cases of \(h = 1\) and \(h = 4\).

\[
\pi_{t+h} = \beta_1 + \beta_2 \pi_{t-1} + \epsilon_t
\]

In line with the above mentioned studies, we will use the ratios of mean square errors to quantify differences between models in equations 5 and 6. In our definition, when the value of this ratio would be less than 1, the model with output gap would beat the simple autoregressive model; when the ratio would exceed 1, using the output gap would in fact decrease the relative ability of the model to forecast inflation.

For determining whether to include output gaps in the model, it is necessary to point out that hard data are of course available only up to the time when a forecast is made. Therefore, we will construct separate time series for output gaps using data only up to given quarters, and also estimate equations 5 and 6 using information up to these quarters. An ideal solution would be to carry out calculations using
data valid at a selected point in time, as undertaken by Clausen a Clausen (2010). However, there exists no such database of past data for the four countries we examine. We therefore use currently valid data, but use floating end of time series.

We will consider four setups of models:

- **Real-time estimate**: output gap time series and equation parameters are estimated using data available up to a given quarter.

- **Real-time estimate, final output gap**: parameters are again estimated using available data up to a given quarter, but we would simulate that "final" and "true" values of output gaps are known, i.e. those calculated as if in Q4/2012.

- **Final estimate**: equations parameters are fixed at their "final" value, i.e. with all available data until Q4/2012. Output gaps are also calculated using all available data.

- **Final estimate, constant tax rates**: the methodology is the same as in the previous setup, but instead of HICP inflation, we use HICP adjusted for changes in indirect tax rates, as published by Eurostat.

Table 1 presents the results of all models setups, i.e. ratios of mean square errors of inflation forecasts. We can see that in most cases, models which include output gap show superior results to autoregressive models. It is not surprising that differences in relative performance of models diminish with increasing forecast horizon (t+1 to t+4).

A closer look reveals a quite surprising result: using "final" values of output gap does not necessarily improve forecasting performance of models. The most prominent example is Slovakia and HP filter-based output gap: when final estimates are used, performance worsens from 0.89 to 0.98. The opposite example is Poland, where final estimates greatly improves forecasting capabilities of models. We can also see that with the exception of Slovakia, using production function improves forecasts especially in real time, which may be caused by the use of relatively large amount of data, which indeed helps to improve output gap accuracy.

Despite the relatively common indirect tax rate changes in these four countries, it is still possible to improve forecasts using output gap when using the HICP inflation. However, in Hungary, the tax changes are so distortive to the original HICP series that differences between models are quite small. On the other hand, using a time series adjusted for these changes is quite desirable in the case of the Hungarian economy: including output gap then greatly improves adjusted inflation forecasts, again as compared to a simple autoregressive model.

<table>
<thead>
<tr>
<th>Czech Republic</th>
<th>HP vs. AR(1) t+1</th>
<th>HP vs. AR(1) t+4</th>
<th>PF vs. AR(1) t+1</th>
<th>PF vs. AR(1) t+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-time estimate</td>
<td>0.90</td>
<td>0.96</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>Real-time estimate, final output gap</td>
<td>0.88</td>
<td>0.97</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>Final estimate</td>
<td>0.85</td>
<td>0.94</td>
<td>0.84</td>
<td>0.93</td>
</tr>
</tbody>
</table>
5 Possible model improvements and comparison of results

In general, we can see that for the Czech Republic, Slovakia, Poland and Hungary, output gap estimated by means of the HP filter and production function is relatively suitable for inflation forecasting. A possible next step would be to find such designs of output gap that would yield even better results. As hinted in previous sections, there are various other methods, let it be structural VAR models, several multivariate methods as multivariate HP filter, multivariate Beveridge-Nelson decomposition. It is also possible to think of a different setup of the production function or its components (NAIRU, capital stock).

Secondly, it is possible to improve the forecasting model themselves: using a simple Phillips curve may not be enough to thoroughly describe inflation development. There are other variables that may have an impact on inflation, e.g. unit labour costs, exchange rate, inflation expectations etc. Coe a McDermott (1997) presented a model describing inflation using not only output gap, but also money supply. According to the authors, this variable influences inflation indirectly, by means of inflation expectations.

We are not aware of any study describing comparison of inflation forecasts models for the four selected economies. Nevertheless, similar results were found in analyses describing different economies. Clausen and Clausen (2010) examined the ability of output gap to forecast inflation in the USA, United Kingdom and Germany and found out that the two selected methodologies (HP filter and linear trend) yield plausible results. Hjelm and Jönsson (2010) analysed properties of output gaps in Sweden using several methodologies, and also reached acceptable results for some of them, especially for several types of multivariate unobserved components models.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t+1</td>
<td>t+4</td>
<td>t+1</td>
<td>t+4</td>
</tr>
<tr>
<td><strong>Final estimate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>constant tax rates</strong></td>
<td>0.87</td>
<td>0.94</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Slovakia</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real-time estimate</td>
<td>0.89</td>
<td><strong>1.01</strong></td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>Real-time estimate, final output gap</td>
<td>0.98</td>
<td><strong>1.03</strong></td>
<td>0.98</td>
<td><strong>1.02</strong></td>
</tr>
<tr>
<td>Final estimate</td>
<td>0.98</td>
<td><strong>1.02</strong></td>
<td>0.98</td>
<td><strong>1.02</strong></td>
</tr>
<tr>
<td>Final estimate, constant tax rates</td>
<td>0.97</td>
<td><strong>1.02</strong></td>
<td>0.97</td>
<td><strong>1.01</strong></td>
</tr>
<tr>
<td><strong>Poland</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real-time estimate</td>
<td>0.98</td>
<td>0.88</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Real-time estimate, final output gap</td>
<td>0.79</td>
<td>0.85</td>
<td>0.91</td>
<td>0.95</td>
</tr>
<tr>
<td>Final estimate</td>
<td>0.78</td>
<td>0.86</td>
<td>0.84</td>
<td>0.91</td>
</tr>
<tr>
<td>Final estimate, constant tax rates</td>
<td>0.88</td>
<td>0.91</td>
<td>0.84</td>
<td>0.89</td>
</tr>
<tr>
<td><strong>Hungary</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real-time estimate</td>
<td>0.97</td>
<td><strong>1.02</strong></td>
<td>0.93</td>
<td><strong>1.02</strong></td>
</tr>
<tr>
<td>Real-time estimate, final output gap</td>
<td><strong>1.00</strong></td>
<td><strong>1.08</strong></td>
<td><strong>1.00</strong></td>
<td><strong>1.08</strong></td>
</tr>
<tr>
<td>Final estimate</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Final estimate, constant tax rates</td>
<td>0.81</td>
<td>0.94</td>
<td>0.80</td>
<td>0.96</td>
</tr>
</tbody>
</table>
6 Conclusion

Many central banks use the concept of output gap to forecast inflation, although this variable is unobserved and subject to many methodological issues. The aim of this paper was to find out forecasting abilities of output gap in the Czech Republic, Slovakia, Hungary and Poland. The analysis itself was conducted on both real-time and ex-post data and two methods were utilised to calculate the output gap: the HP filter and the production function. These two were chosen out of a large array of other methods primarily due to their current mainstream use in many national and international institutions. To quantify the forecasting ability, a simple Phillips curve model was compared to an autoregressive model using ratio of mean square errors.

Our analysis confirmed that at a general level, including output gap into models can improve their forecasting capabilities. Nevertheless, it is not a general rule that the use of "final" output gap estimates is superior to those in real time, as was demonstrated in the case of the Slovak economy. In Hungary, where there have been frequent changes in indirect tax rates, it is better to use HICP adjusted for these influences.

The results presented in this paper are not final and can be improved in several ways. Output gap can be calculated using different methods that may yield more accurate inflation forecasts. Inflation models can be enhanced by more relevant variables - as output gap is not the only variable having and impact on inflation - let it be inflation expectations, unit labour costs or exchange rates.

References


---

1 The NAIRU (Non-accelerating Inflation Rate of Unemployment) is again an unobserved variable and its estimation is subject to similar problems as in the case of the potential product. For more discussion see e.g. Gordon (1997).

2 Due to the volatility of quarterly data on hours worked, we will use employment (headcount) from the Labour Force Survey.

3 In this study, we are interested only in the level of potential output. The production function presented here is only marginally sensitive to changes in alfa parameter. When changing this parameter, only contributions of capital, labour and TFP changes, not the overall level of potential product.

4 According to several studies (eg. Hjelm and Jönsson, 2010), the very ability of the output gap to forecast inflation is one of important criteria that determine whether a particular output gap methodology is suitable for a particular economy.

5 For the sake of simplicity, we use only one time lag, as in Clausen and Clausen (2010). A possible alternative approach would be to chose the number of lags on the basis of information criteria.

6 Values above 1, when it is advantageous to use autoregressive models, are shown in bold.

7 We used data from Eurostat ranging from Q1/1998 to Q4/2012. The comparison of models is undertaken on the time span of Q1/2005 to Q4/2012. Due to relatively short time series of HICP adjusted for tax changes, in this setup, we only use “final estimate” methodology and analyse the time span of Q1/2004 to Q4/2012.