ESTIMATING OUTPUT GAP AND POTENTIAL OUTPUT FOR RUSSIA AND ITS USEFULNESS BY FORECASTING INFLATION

Abstract:
This paper deals with an estimation of output gap and potential output for Russian’s economy. Three methods of estimation have been used for estimating these two unobservable variables: Hodrick-Prescott filter, production function and SVAR model. All methods of estimation showed very similar course, although obtained values were not identical. Then obtained values of the output gap were used to analyse the ability of output gap to forecast inflation. Two simple gap models were used for this purpose. The results showed that output gap could be used as useful indicator if inflation, according to all methods of estimation output gap.

Keywords:
output gap, HP filter, SVAR model, production function, Kalman filter, inflation

JEL Classification: C53, E31, E32
Introduction

Many central banks and international institutions use output gap, beside other things, for its ability to forecast inflation despite of generally accepted facts, which most economic researches surprisingly agree with, that it is impossible to estimate potential output or output gap with definite precision, and agree more with the term „estimation“. Results of particular estimations are used then in so called gap models, which test ability of output gap to forecast inflation. It should hold true, that when economy is overheating, when real output is above potential output, there should be inflationary pressures in the economy and inflation should increase. On the contrary, when real output is under its potential level, there should be deflationary pressures in the economy and inflation should go down.

How it has been said, problems with research of ability of output gap to forecast inflation start already by the estimates of output gap themselves. These variables are not directly observable and they are not possible to measure them with absolute accuracy. It is generally recommended to speak more about „estimation“ than „measurement“.

Output gap in this paper will be defined as deviation of real output from potential output. If real output is above potential output, there will be a positive output gap in economy, and on the contrary, if real output is under potential output, there will be a negative output gap in the economy. Potential output in this paper will be understood according to de Masi (1997) and will mean the highest possible level of product, which is produced by maximum utilization of resources of production which do not cause inflationary pressures.

For estimating potential output and output gap for Russian´s economy, there will be used some mainstream methods: Hodrick-Prescott filter (1997) (HP), production filter and structural VAR model. Hypothesis, that results of particular estimations confirm ambiguity of measurement of output gap and potential output.

The gained values of previous measurements will be used subsequently for estimating ability of output gap to forecast inflation. For this purpose, a simple gap model according to Coe and McDermott (1997) will be used, where versions with level of output gap and its first differences will be used, too. Both versions of gap model
will confirm the hypothesis, that output gap for Russian’s economy is useful for estimation inflation. 

Because there is very little research of estimation output gap and potential output for Russian’s economy, and its ability to forecast inflation, too, the author of this paper sees in this the biggest value added of this paper.

2. Previous research

If the author of this paper is well-informed, there is very little research dealing with Russian’s output gap or potential output. There are also few institutions dealing with estimating these two unobservable variables. IMF does not publish data about output gap or potential output, only OECD.¹ It is possible to mention a study from Oomes and Dynnikova (2006), which estimated output gap for Russian’s economy with production function. Hanson (2009) dealt with forecasting development of Russian’s economy generally until 2020. There is no known study or paper dealing with examination of ability of output gap to forecast inflation. Therefore gap model according to Coe and McDermott (1997) will be used for examination of ability of output gap to forecast inflation. The authors applied their model on selected economies of south-eastern Asia and Australia. Other studies dealing with these problems, but for other economies, are for example studies from Claus (2000) (for economy of New Zealand) or Menyhért (2008) (for economy of Hungary). In the lack of this research for Russian’s economy, the author of this paper sees the main value added of this paper. The aim of this paper is then to fill up this lack in this research.

3. Data

All data used for estimates of output gap for Russian’s economy are from Statistical Office for Russian’s Federation. The length of time period was for all estimates the same and was from 1995Q1²-2012Q3, in quarterly intervals. The length of this time period has been selected because of accessibility and credibility of data necessary for

¹ OECD publish data in annual intervals only, not quarterly.
estimations themselves. By some methods, time series were prolonged by predictions due to more precisely estimations.

4. Estimates of potential output and output gap according the selected methods

Two mainstream methods were selected for estimates of output gap and potential output: Hodrick-Prescott filter and production function. One structural methods was added to these two well-known methods for improvement of credibility of estimates: structural VAR model, or SVAR.

4.1 Hodrick-Prescott filter

Hodrick-Prescott filter (1997) (Hodrick a Prescott (1997)) belongs among the most frequently used methods of estimation output gap and potential output, and it is used by many national or international institutions. To the most frequently mentioned advantages of this method of estimation belongs ease of input data, when only time series of GDP is necessary. Easy mechanical application accessible in most of the econometrical software belongs to the other advantages.

By this method, time series of GDP can be decomposed on its cyclical component \( c_t \) and trend component (represents potential growth) \( g_t \). It is possible to write:

\[
y_t = g_t + c_t
\]

(1)

Trend component \( g_t \) is obtained by minimising the equation 2, if sum of squares of cyclical component is minimalized, which is also penalised by deviations in second differences of trend component:

\[
\min_{\{g_t\}} \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2
\]

(2)

where \( \lambda \) is parameter of smoothness, which penalises variability of trend component. The higher values of parameter of smoothness mean more smoothed potential component and more volatile cyclical component. For our purpose, we have determined \( \lambda = 1600 \).
On the other hand, this method of estimating potential output and output gap has some disadvantages, too. Except its strictly statistical character which abstracts from any economic theory, one of the big disadvantages is problem with determining of parameter of smoothness. This parameter is set outside the model and according to many studies, there is endless debate which values are the best ones. Most studies accept values from Hodrick and Prescott (1997), who recommended $\lambda = 100$ for annual data, $\lambda = 1600$ for quarterly data and $\lambda = 14400$ for daily data. But many economists haven’t agreed with this solution, e.g. Coe a McDermott (1997) haven’t recommended these values, because they have been set for USA and setting these values for other, different economies could lead to substantial misrepresentation of results of estimates. Another problem is with value at both ends of time series, known as end point bias. Commonly accepted and by economists used solution is to prolong time series. Time series in this paper has been prolonged too, with the forecast for next two years.

Results of estimates of potential output $v$ billions of euros and of output gap in percent of potential output are shown in figure 1a and 1b.

**Figure 1: Potential output and output gap: HP filter**

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3 For further possibilities of solving of this problem, see other studies, e.g., Cooley & Ohanian (1991) and Correia, Neves & Rebelo (1992), Baxter & King (1999) or Ravn & Uhlig (2002).
4.2 Production function

Strictly statistical character of previous method of estimation potential output and output gap does not contain any economic theory which could explain more precisely the course of these two variables. This economic theory contains a structural method of estimation: production function. This method belongs beside HP filter among the most frequently used methods of estimation of output gap and potential output, e.g. is used by OECD (see e.g. Beffy, Richardson & Sedilliot (2007)) or EC (see e.g. Hjelm & Jönsson (2010)).

In this paper, Cobb-Douglas production function with constant returns of scale and Hicks neutral technological progress was selected for estimation. Likewise HP filter, time series of real output in time $t (Y_t)$ is decomposed into components: employment
$L_t$, capital stock $K_t$ and total factor productivity $A_t$. According to Report of Ministry of Finance of Russian’s Federation (2013) is labour share on income for Russian’s economy approximately 50% and therefore parameters $\alpha$ and $\beta$ have following values: $\alpha = 0.50 \text{ and } \beta = 0.50$.

For real output can be written:

$$Y_t = A_t \cdot K_t^\alpha \cdot L_t^{1-\alpha} \quad (3)$$

Before the own estimation potential output, values of single potential variables will be estimated. Potential capital stock will be obtained according to well-established standards (see e.g. CBO (2001)) when the capital stock is identified with actual capital stock. The actual capital stock is calculates as sum of current fixed investments of past capital stock adjusted for depreciation. Rate of depreciation was determined according to well-established standards (see e.g. Mourre (2009) to 0.05.

Another potential component which is necessary for estimation potential output is potential employment. It is divided into three components: working age population, i.e. 15-64 years, trend participation rate which will be obtained with using HP filter and NAIRU.4 We can write for potential employment:

$$L_t^* = \text{pop}_{15-64} \cdot \text{part}_{t} \cdot (1 - \text{NAIRU}_t) \quad (4)$$

The last potential component is potential TFP.5 It will be defined as trend TFP, again with using HP filter.6

Results of estimation of potential output and output gap for Russian’s economy are shown in figure 2. They will be compared with other methods of estimation later.

4 NAIRU – non-accelerating inflation rate of unemployment
5 TFP – total factor of productivity
6 Baxter-King filter is used sometimes.
Figure 2: Potential output and output gap: production function

Source: Statistical office, own estimates
4.3 SVAR model

A relative sophisticated and quite often used method of estimation potential output and output is gap is structural VAR model or SVAR. These models, first introduced by economists Blanchard & Quah (1989), contain not only robust statistical framework but some economic restrictions that help to explain the course of output gap more precisely. The basic assumption is a division of real output into three components: deterministic trend, shocks which influence supply side of economy and transitory shocks who influence demand side of economy. Deterministic trend and shocks that influence supply side of economy represent potential product while transitory shocks represent cyclical component, output gap. It is important to distinguish between supply and demand shocks, but only supply shocks have influences on output in long run. Demand shocks can influence economy only in short run.

Procedure of bivariate SVAR model with HDP and inflation is as follows:
Under stationary vector $Z$ will be understood $n \times 1$ stationary vector with $n_{1\star}$ vector $I(0)$ variables a $n_{2\star}$ vector $I(1)$ variables, that it will hold $Z_t = (\Delta X'_{1t}, X'_{2t})$. Afterwards, stationary vector $Z_t$ can be written with application of Wold decomposition theorem in its reduced form:

$$Z_z = \delta(t) + C(L)\varepsilon_z$$  \hspace{1cm} (5)

where $\delta(t)$ is deterministic variable, $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is matrix of polynomial lags, $C_0 = I_n$ matrix of identity, vector $\varepsilon_z$ will be forecast of errors in $Z_z$ informing about lag values of $Z_z$, while its mean value will be zero and $E(\varepsilon_z, \varepsilon_z) = \Omega$ with positively defined $\Omega$. Equation(6) is subsequently decomposed into permanent and transitory components:

$$Z_z = \delta(t) + C(1)\varepsilon_z + C^*(L)\varepsilon_z$$  \hspace{1cm} (6)

where $C(1) = \sum_{i=0}^{\infty} C_i$ a $C^*(L) = C(L) - C(1)$. Then is defined $C_t(1)$ as permanent multiplicator of vector $X_{1t}$. According to LRRO approach $Z_z$ will have a following structural form:

$$Z_z = \delta(t) + \Gamma(L)\varepsilon_z.$$  \hspace{1cm} (7)
where \( \eta \) is \( n \)-vector of structural shocks with non-zero mean value and \( E(\eta_i, \eta_j) = I_n \). This structural form is possible to obtain from the equation (6) and exploiting of relationship

\[
\Gamma_0 \Gamma_0' = \Omega = \Gamma_0 \eta \ a \ C(L) = \Gamma_L \Gamma_0^{-2}. \tag{8}
\]

Subsequently long-term covariance matrix in reduced form is obtained from equations (7) and (8)

\[
C(1)^{\Omega}C(1) = \Gamma(1)\Gamma(1)', \tag{9}
\]

Finally is matrix with long-term restrictions of covariance matrix in structural form derived from the procedure mentioned above.

There are relative many modifications of SVAR models.\(^7\) For our purpose, bivariate SVAR model with real output and inflation according to Robertson & Wiskens (1997) was selected. As the restriction was determined that nominal shock has no effect on GPD. Only supply shock can influence GDP in long run.

Two variables were included in reduced bivariate SVAR model:

\[
\Delta y_c = \beta_1 + \psi_{11}(L)\Delta y_{c-1} + \psi_{12}(L)\Delta \pi_{t-1} + \mu_{\Delta y_c}, \tag{10}
\]

\[
\Delta \pi = \beta_2 + \psi_{21}(L)\Delta y_{c-1} + \psi_{22}(L)\Delta \pi_{t-1} + \mu_{\Delta \pi_c}, \tag{11}
\]

where \( \Delta y_c \) is difference (logarithm) HDP, \( \Delta \pi \) is difference of inflation and \( \mu_{\Delta y_c} \) are unexplained shocks in the model. Subsequently, by meeting of assumptions of

\(^7\) Blanchard & Quah (1989) analyzed bivariate SVAR model with product and unemployment, where unemployment will have no long-term effect on product, Camba-Mendez & Rodriguez-Palenzuela (2003) extended the model to trivariate model with inflation, which will have no long-term effect on product, too. Fourvariate SVAR model was analyzed by Hjelma & Jönsson (2010), who used product, inflation, unemployment and labour productivity. Fabiani (2001) estimated output gap even with an five variate SVAR modele, where variables were real wage, output, inflation, unemployment and share of labor wages on income.
realization of structural shocks budou are variables in the model defined as a sum of current and past shocks:

\[ \Delta y_t^c = S_{11}(L)v_t^e + S_{12}(L)v_t^D \]  
(12)

\[ \Delta y_t^e = S_{21}(L)v_t^e + S_{22}(L)v_t^D \]  
(13)

Finally the change of product attributed to potential output is defined as follows:

\[ \Delta y_t^p = S_{11}(L)v_t^e \]  
(14)

The change of product attributed to its cyclical component (including demand shocks with no long-term effects on product):

\[ \Delta y_t^c = S_{12}(L)v_t^D \]  
(15)

Output gap and potential output for Russian’s economy estimated with bivariate SVAR model with GDP and inflation are shown in following picture.

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8 By SVAR2 model is used model in form 2x2 (by SVAR3 model would be form of 3x3 used).
Figure 3: Production function and potential output: SVAR model

Source: Statistical office, own estimates
5. Comparison of estimates

Comparison of estimated values with data from OECD brings table 1. It is possible to see, that although data show a similar course, the values are not identical.

Table 1: Own estimates and OECD: a comparison

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>HP filter</td>
<td>-5,5</td>
<td>0,1</td>
<td>-4,2</td>
<td>0,7</td>
</tr>
<tr>
<td>SVAR</td>
<td>-3,5</td>
<td>0,7</td>
<td>-4,6</td>
<td>1,0</td>
</tr>
<tr>
<td>Production function</td>
<td>-7,1</td>
<td>-0,6</td>
<td>-3,5</td>
<td>0,4</td>
</tr>
<tr>
<td>OECD</td>
<td>-4,8</td>
<td>0,2</td>
<td>-3,9</td>
<td>-0,4</td>
</tr>
</tbody>
</table>

Source: Statistical office, own estimates

Single estimates of output gap according to selected methods compares figure 4. It is possible to see, that single methods of estimation have very similar course, but there were not identical values for the same time period. Hypothesis about impossibility of measurement of output gap with absolute accuracy was rejected.
6. Output gap and inflation

Although output gap is not possible to measure with absolute reliability, but more only to estimate, it has been shown, that it is possible to use it for forecasting inflation, e.g. according to some studies, e.g. Coe a McDermott (1997) or Claus (2000).

Two gap models according to Coe and McDermott (1997) will be used for confirmation or rejection of hypothesis about ability of output gap to forecast inflation. The first model will analyse relationship between level of output gap and change in inflation:

\[ \pi_t = \alpha + \sum_{k=0}^{p} \beta_k \text{gap}_{t-k} + \varepsilon_{t} \]

(16)

under \( \pi_t \) will be understood logarithmic difference of CPI, \( \text{gap}_{t-k} \) logarithmic difference between actual and potential output and \( \varepsilon_{t} \) random variable.
Second model will analyse relationship between change in inflation and change in output gap:

\[
\Delta \pi_t = \alpha_2 + \sum_{i=3}^{T} \beta_{2k} \Delta \text{gap}_{t-k} + \epsilon_{2t}
\]

where \( \Delta \pi_t \) is logarithmic difference of, \( \Delta \text{gap}_{t-k} \) logarithmic difference of actual and potential product and \( \epsilon_{2t} \) is random variable, \( \Delta \) means an operator of first difference.

Following hypothesis will be defined in the gap models: if sum of coefficients \( \beta_{1k} \) is positive and statistically significant, positive output gap will lead to increase of inflation. Results of testing of both models brings table 2. By both models, optimal lag length was determined using Schwarz information criterions (SIC) and was set on three. All signs of single coefficients relate to these lags. All F-tests confirmed the ability of output gap to forecast inflation on 5% level of significance. Statistics \( R^2 \) indicates that the model with the change in output gap explains changes in inflation better. Sum of coefficients \( \beta_{1k} \) was in all cases positive and statistically significant.

### Table 2: The gap models

<table>
<thead>
<tr>
<th>O. gap</th>
<th>Lags</th>
<th>Sum of koef. beta</th>
<th>sign</th>
<th>F-test</th>
<th>R2</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP</td>
<td>3</td>
<td>0,072</td>
<td>+++</td>
<td>3,76**</td>
<td>0,18</td>
<td>0,167**</td>
</tr>
<tr>
<td>PF</td>
<td>3</td>
<td>0,0043</td>
<td>--&gt;</td>
<td>5,03**</td>
<td>0,23</td>
<td>0,452**</td>
</tr>
<tr>
<td>SVAR</td>
<td>3</td>
<td>0,0021</td>
<td>+++</td>
<td>2,1**</td>
<td>0,32</td>
<td>0,154**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>O. gap</th>
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<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP</td>
<td>3</td>
<td>0,063</td>
<td>++</td>
<td>4,98**</td>
<td>0,22</td>
<td>0,521**</td>
</tr>
<tr>
<td>PF</td>
<td>3</td>
<td>0,0038</td>
<td>++-</td>
<td>5,53**</td>
<td>0,32</td>
<td>0,745**</td>
</tr>
<tr>
<td>SVAR</td>
<td>3</td>
<td>0,0012</td>
<td>++-</td>
<td>3,8**</td>
<td>0,33</td>
<td>0,543**</td>
</tr>
</tbody>
</table>

Source: Statistical office, own estimates
7. Evaluation of results and possibility of further research

Single estimates of output gap for Russian´s economy showed, that it is impossible to measure output gap for this economy with an absolute accuracy, too. It is better to speak about “an estimation” than “a measurement”. Despite of this fact, gap models showed that it is possible to consider output gap as useful indicator of inflation. Unfortunately, a comparison with previous research is impossible, because there is no known study dealing with output gap as indicator of inflation for Russian´s economy. On the other hand, improvements of gap models are possible, too. One of the possible improvements of the model could be insertion of additional variables into the gap model, e.g. money demand. All of these arrangements could improve the ability of output gap to explain inflation in an economy.

Conclusion

The aim of this paper was to deal with potential output and output gap for Russian´s economy. Three methods for estimating these two unobservable variables was selected: HP filter, production function and SVAR model. Given methods of estimation were estimates for time period 1995Q1-2012Q3. All methods of estimation estimated very similar course of output gap, although particular methods themselves didn’t achieve the same values for the same time period. All results were achieved in the case of comparison with data from OECD after re-counting own estimates on values in annual intervals.

Then ability of output gap to forecast inflation was analysed. It should hold that if there is a positive output gap in economy, inflation should increase and on the contrary, in the case of a negative output gap, inflation should decrease. Two gap models for confirmation or rejection of this relationship was selected. First model analysed relationship between change in inflation and level of output gap, and the second model analysed relationship between change in inflation and change in output gap.

Both models confirmed the hypothesis, that output gap is an useful indicator of inflation, where better results were achieved by gap model with the change in output gap. According the models, the best results should be given by SVAR model as the most sophisticated method from the three selected methods of estimation.
Naturally, other results could have been given, if output gap would had been estimated with other, more sophisticated methods of estimation, e.g. with a method from group of multivariate methods of estimation. Suitable modifications of gap models themselves could bring other results, too, e.g. extension by money demand. These problems could stay as one of many other possibilities how to make the gap models better and more relievable. This paper tried to fill up the lack in research on this field for Russian’s economy, although naturally other research is not only possible, but suitable, too.

References


Министерство финансов Российской Федерации (Ministry of Finance of the Russian Federation), 2013.

