Macro-Based Parametric Asset Allocation

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Abstract

This paper presents a novel approach to asset allocation which builds up on macroeconomic factors. Without doubt the financial return of asset classes are interlinked with the economy. However, it is not that clear how to bring the finance and economy world together within a portfolio's asset allocation. I propose a direct modeling of the weights with global macroeconomic risk factors. These risk factors are not asset class specific but potentially related to the return of all asset classes. In this paper I focus on three asset classes: stocks, bonds and the risk free asset. The approach is robust, links macroeconomic factors to financial returns intuitively and outperforms a standard 60/40 portfolio almost twice in terms of the Sharpe Ratio - in sample and out of sample. This outperformance even remains to a large extent when considering transaction or leverage costs.

Keywords: portfolio management; asset allocation; macro based; parametric weights

JEL Codes: G10, G11, G17

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1 Introduction

Without doubt there is numerous evidence that the financial return of asset classes such as stocks and bonds are interlinked with the economy. However, it is not that clear how to bring both worlds together in an portfolio's asset allocation.

By far the most prominent models in asset allocation are the models of Markowitz (1952) and Black and Litterman (1992). In the approach of Markowitz (1952) asset class returns and a covariance matrix related to these returns need to be estimated for the asset allocation process. This induces the danger of adding up estimation errors and often results in an unstable asset allocation. Moreover, Jacobs, Müller and Weber (2010) show that a simple heuristic approach can outperform Markowitz and its variations cost-efficiently.

In Black and Litterman (1992) the investor updates the information implicitly revealed by the market with his own return expectations. Returns are usually assumed to follow a normal distribution and as in Markowitz (1952) there is no obvious link between the economy and the model's proposed asset allocation.

In both approaches the investor needs to estimate the asset classes' expected returns. This is difficult and often subjective. Therefore I suggest a macro based parametric asset allocation approach using a method proposed by Brandt, Santa-Clara and Valkanov (2009) . Different to the original paper I do not consider the allocation of stocks within a stock portfolio but focus on asset allocation. The weights are directly estimated through global risk factors. Hence, there is no need to estimate expected returns.

The idea is that global risk factors drive the performance of asset classes and are important for each asset class - in absolute terms or relatively. The challenge is to identify those economic forces that cause expected returns and hence the weights to change. This approach implicitly reveals capital market dynamics and how they translate into portfolio weights. Changes of portfolio weights are derived directly as triggered by changes in the global risk factors. Naturally, the importance or "loading" of each risk factor could be different between asset classes. Examples for global risk factors are the ted spread or the term spread which can both be interpreted as indicators for the health of the economy and indicators for the expected performance of financial markets.

A similar approach relying on the method of Brandt, Santa-Clara and Valkanov and global risk factors was followed by Herrero and Herrero (2009) who construct a hedge fund strategy consisting of up to 46 long/short portfolios of various asset class pairs. From a portfolio perspective this is a different approach to the one presented here as the asset allocation is not considered directly but long-short-subportfolios are built and optimized separately. These are then combined in a so called expected loss exceeding value at risk methodology. A thorough out of sample and robustness discussion is missing.

In this paper I focus on three asset classes: stocks, bonds and the risk free asset. The results of the method suggested are promising. The strategy outperforms its benchmark, a standard 60/40 stock-bond-portfolio, significantly - the Sharpe Ratio is almost doubled. Jensen's Alpha measured relatively to the benchmark is positive and almost always significant apart from some model specifications when ρ gets large. The statistically significant global risk factors have meaningful signs and are intuitive to interpret. The model's outperformance prevails when considering transaction costs or leverage costs and an application of the model is straight forward due to stable and robust estimates of parameters and reasonable asset allocation weights. Although the ambition is not to find an asset allocation tool which a portfolio manager follows unquestioned, the suggested asset allocation approach in this paper can add significant value to the tactical asset allocation process in terms of providing a framework to the asset manager.

The paper proceeds as follows. In section 2 I describe the methodology and idea behind the model. Section 3 discusses the statistical approach. For estimation I use generalized methods of moments and an iterative optimization routine which averts local maxima.

A way of testing whether the methodology and statistical approach is appropriate to use for the asset allocation problem is to verify the proposed method on well known examples of the literature. Therefore I compare the analytic results with the results of the parametric portfolio approach. For this I discuss Merton (1969) and Campbell and Viceira (1999) in section 4.

Since a large sample of data is crucial for the methodology to work I constrain myself to US data and take the results as proxy for a global approach. The data is described in detail in section 5. In section 6 results are presented and three models are discussed in detail: a value and two momentum models.

In section 7 I include transaction and leverage costs to the model. As the definition of the out of sample period is up to some degree arbitrary I perform several robustness checks in section 8 to verify the performance of the model. Finally, section 9 provides a summary of the paper.

2 Idea and Methodology

At each date t there is a fixed number of investable risky asset classes, N. The investor faces the problem of allocating his funds among these asset classes at each point in time such that his conditional expected utility of the portfolio returns is maximized. The percentage allocation of his funds to asset class i at time t is denoted as $w_{i,t}$. Each asset class i has a return $r_{i,t+1}$ measured from t to $t+1$. Similarly, the portfolio return is $r_{p,t+1}$. Suppose the return of these asset classes is associated with a vector of K risk factors, x_t

observed at date t. Assume the investor's utility function $u(.)$ is time separable. Then the investor faces the following optimization problem

$$\max_{\{w_{i,t}\}_{i=1}^{N}} \mathbb{E}_t \left[u \left(\sum_{i=1}^{N} w_{i,t} r_{i,t+1} \right) \right] \tag{1}$$

In contrast to Brandt, Santa-Clara and Valkanov (2009) the investor is not interested in allocating his funds within a pure stock portfolio but in allocating his funds among asset classes such as stocks and bonds. The asset class weights are parameterized by a function $f(.)$ which depends on the risk factors x_t. Hence, not the returns, but the weight given to the assets are directly related to the risk factors.

$$w_{i,t} = f(x_t; \theta_i) \tag{2}$$

It is important to observe that the k'th macroeconomic risk factor $x_{k,t}$ is the same for each asset class i. However, the parameters $\theta_{i,k}$ with which this risk factor "loads" on the weight $w_{i,t}$ of asset i can be different between asset classes. As in Brandt, Santa-Clara and Valkanov (2009) suppose that the weighting function is linear

$$w_{i,t} = \bar{w}_{i,t} + \theta_i' \hat{x}_t \tag{3}$$

$\bar{w}_{i,t}$ are benchmark weights and $\theta_i' \hat{x}_t$ captures the deviation to these benchmark weights. To ensure that the portfolio weights sum up to one a risk free asset with return $r_{f,t+1}$ from t to $t+1$ at which the investor can borrow and lend serves as residual. The return of the risk free asset is known at t. There is no leverage constraint in place, hence the investor could borrow unboundedly. This is implicitly constrained later by introducing leverage costs. Formally, at any time t the following condition must hold:

$$\sum_{i=1}^{N} w_{i,t} + w_{r_f,t} = 1 \tag{4}$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} w_{i,t} \equiv 1 - \sum_{i=1}^{N} f(x_t; \theta_i)$ is the percentage weight of the risk free asset at time t. This is similar to the condition that the $k'th$ coefficient of the risk free asset $\theta_{r_f,k} = -\sum_{i=1}^{N} \theta_{i,k}$. The utility maximization problem of the investor can then be written as

$$\max_{\{w_{i,t}\}_{i=1}^{N}} \mathbb{E}_t \left[u \left(\sum_{i=1}^{N} f(x_t; \theta_i) r_{i,t+1} + w_{r_f,t} r_{f,t+1} \right) \right] \tag{5}$$

As the coefficients are constant they maximize the investor's conditional expected utility not only at one given date but for all dates. Therefore the coefficients maximize the investor's unconditional expected utility. However, the coefficients are not constant across assets. Hence, the weight of each asset does not necessarily only depend on its risk factors but could also depend on the asset class' historic return. I account for this by considering the asset class' past return as a risk factor specific to the asset class.

This implies that the optimization problem (5) can be rewritten as the following unconditional optimization problem with respect to the coefficients θ_i

$$\max_{\theta_i} \mathbb{E}[u(r_{p,t+1})] = \mathbb{E}\left[u\left(\sum_{i=1}^{N} f(x_t;\theta_i) r_{i,t+1} + w_{r_f,t} r_{f,t+1}\right)\right] \tag{6}$$

Finally, the sample analog is

$$\max_{\theta_i} \frac{1}{T}\sum_{t=0}^{T-1}[u(r_{p,t+1})] = \sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{N} f(x_t;\theta_i) r_{i,t+1} + w_{r_f,t} r_{f,t+1}\right)\right] \tag{7}$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} f(x_t;\theta_i)$. In the linear policy case the optimization problem is

$$\max_{\theta_i} \frac{1}{T}\sum_{t=0}^{T-1}[u(r_{p,t+1})] = \sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{N} \left(\bar{w}_{i,t} + \theta_i'\hat{x}_t\right) r_{i,t+1} + w_{r_f,t} r_{f,t+1}\right)\right] \tag{8}$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} w_{i,t}$.

I assume constant relative risk aversion utility with risk aversion parameter ρ

$$u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\rho}}{1-\rho} \tag{9}$$

3 Statistical Approach

In order to maximize the investor's utility the estimates $\hat{\theta}_i$ should satisfy the first order conditions of the maximization problem (equation 8) for each parameter $\theta_{i,k}$ of each risky asset class i and risk factor k. Thus $N \times K$ coefficients are estimated where N is the number of risky assets and K the number of characteristics. The first order condition of parameter $\theta_{i,k}$ is

$$\frac{1}{T}\sum_{t=0}^{T-1} h(r_{t+1}, x_t; \theta_{k,i}) \equiv \frac{1}{T}\sum_{t=0}^{T-1} u'(r_{p,t+1})\left(\hat{x}_{k,t}(r_{i,t+1} - r_{f,t+1})\right) = 0 \tag{10}$$

with

$$u'(r_{p,t+1}) = u'\left(1 + \left(\sum_{i=1}^{N}(\bar{w}_{i,t} + \theta_i \hat{x}_t) r_{i,t+1}\right) + w_{r_f,t} r_{f,t+1}\right) \tag{11}$$

where

$$w_{r_f,t} = 1 - \sum_{i=1}^{N}(\bar{w}_{i,t} + \theta_i \hat{x}_t) \tag{12}$$

Following the statistical approach of Brandt, Santa-Clara and Valkanov (2009) the first order conditions can be interpreted as method of moments estimators, specifically as moment conditions. Furthermore the asymptotic covariance matrix of this estimator is (Brandt, Santa-Clara and Valkanov, 2009, and Hansen, 1982)

$$\sum_{\theta} \equiv \mathrm{V}[\hat{\theta}] = \frac{1}{T}[G'V^{-1}G]^{-1} \tag{13}$$

with

$$G = \frac{1}{T}\sum_{t=0}^{T-1} \frac{\delta h_t}{\delta \theta}\bigg|_{\theta=\hat{\theta}} \tag{14}$$

where θ are the stacked estimators. For estimation I use the statistical software R and specifically the package GMM (Chausse, 2011). Since these conditions can not be solved analytically an initial solution guess of θ is required by the numerical algorithm. As with all numerical procedures local maxima could potentially be a problem. A first test on whether there is a local maximum problem is to verify if the solution to the numerical algorithm is independent of the initial solution guess. This test fails clearly. However, this local maxima problem can be solved the following way:

1. In total $K \times N$ coefficients are estimated. For each of these coefficients an initial value is defined. Suppose this initial value is drawn from a pool of m feasible initial solution values for each coefficient.

2. Define the initial solution space with all permutations p of initial coefficient vectors. For example in the case of 4 coefficients and three possible values for each coefficient there are 81 initial coefficient vectors.

3. As either K, N or m gets large the initial solution space grows rapidly and therefore the calculation time to test all possible solutions. Therefore j initial solution vectors are drawn from the initial solution space, where $j \leq p$.

4. Start the optimization procedure with j initial solution vectors. This results in j solutions which should be similar to each other. If they are similar stop the calculation. Else continue:

5. Calculate the first and third quantile value of the solutions for each coefficient.

6. Replace the old initial solution values with the first and third quantile as calculated and the $m-2$ values equally distributed within the first and third quantile. Do this for each coefficient

7. Calculate the j solutions to these initial values. If they are similar, stop. Else do the following loop:

   7.1 Calculate the first and third quantile value of the solutions for each coefficient.

   7.2 If the difference between the first and third quantile is smaller than the values for the first and third quantile of the previous initial solution values replace the previous initial solution values with the corresponding first and third quantile values as calculated and the $m-2$ values equally distributed within the first and third quantile. Do this for each coefficient.

The converged solution of this optimization routine is the best with respect to utility compared to all other and previous solutions. Repeating the optimization with the same or different parameters results in practically the same final solution, i.e. less than a one digit percent deviation between coefficients can occur. This has no influence on the significance or the sign of the coefficients, neither on the return series. I take this as enough evidence to have found the global optimum.

4 Verification of the Method

A way of testing whether the methodology presented is appropriate for the asset allocation problem is to verify the proposed method on well known examples of the literature and compare the analytic results with the results of the parametric portfolio approach. In the following section I discuss two models: Merton (1969) and Campbell and Viceira (1999).

4.1 Merton (1969)

Merton's portfolio problem (1969) states that an investor who lives from time 0 to T decides at each time t about his consumption c_t and how much to invest into a risky asset w_t and how much to invest into a riskless asset $w_{r_f,t}$ earning the risk free rate r_f. These decisions determine the investor's wealth path V_t. The investor's subjective discount rate is δ and his time separable utility function is denoted by $u(.)$. His objective function in continuous time is

$$\max_{w_t, c_t} u(V_0) = \mathbb{E}\left[\int_0^T e^{-\delta s} u(c_s) ds + e^{-\delta T} u(V_T)\right] \tag{15}$$

The investor's wealth evolves according to the following stochastic differential equation

$$dV_t = V_t w_{r_f,t} dt + w_t V_t(\mu dt + \sigma dZ_t - r_f dt) - c_t dt \tag{16}$$

μ is the expected return, σ the stock market volatility and $Z_t \sim N(0, 1)$ is distributed normally. The investor's wealth must be non-negative any time, $W_t \geq 0, \forall t$. Then the risky asset's return $r_{w,t}$ is

$$r_{w,t} = \mu dt + \sigma dZ_t \tag{17}$$

Assuming constant relative risk aversion utility with risk parameter ρ

$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho} \tag{18}$$

the optimal allocation $w_t(V, t)$ to the risky asset can be shown to be time and wealth invariant

$$w_t(V, t) = \frac{1}{\rho}\frac{\mu - r_f}{\sigma^2} \tag{19}$$

Furthermore, the optimal consumption rate is also constant and dependent on the investor's portfolio's value at time t and his time preference rate δ

$$c_t = \delta V_t \tag{20}$$

Transforming the problem into discrete time and assuming $T \to \infty$ the maximization problem becomes

$$\max_{w,c_t} u(V_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} u(c_t)\right] \tag{21}$$

The optimization is subject to the wealth constraint where I assume a constant allocation to the risky asset of w, hence a constant allocation to the riskless asset $w_{r_f,t} = 1 - w$

$$V_t = V_{t-1}\left((1 + r_{w,t})w + (1 + r_f)(1 - w)\right) - c_t \tag{22}$$

where $r_{w,t}$ denotes the return of the risky asset from time $t-1$ to t. This is time consistent to the investor's portfolio wealth V_{t-1} at time $t-1$. Moreover, define V_{t-1}^* to be the wealth after considering the return from the risky and riskless asset but before consumption

$$V_{t-1}^* = (1 + r_{w,t})w + (1 + r_f)(1 - w) \tag{23}$$

Assume furthermore $\rho = 1$ and therefore $u(c_t) = \log(c_t)$ and that consumption is a constant fraction of the investor's portfolio's wealth and his time preference rate as shown by Merton

$$c_t = V_{t-1}^* \delta \tag{24}$$

Hence

$$V_t = V_{t-1}^*(1-\delta) \tag{25}$$

Since $c_t = V_{t-1}^* \delta$ is a constant fraction of wealth the maximization is similar to optimizing V_t and its sample analog is

$$\max_w u(V_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} u(V_t)\right] = \max_w \frac{1}{N} \sum_{n=1}^{N} \left[\sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} u(V_t)\right] \tag{26}$$

For example consider a 100 year period with 50 observations per year and 100 draws and $\mu = 0.05, \sigma = 0.1, \rho = 1 \leftrightarrow u(V_t) = \log(V_t), r = 0.043$. This corresponds to a theoretical optimal allocation to the risky asset of $w = \frac{\mu - r}{\rho \sigma^2} = 0.7$.

The simulation analog where I optimize for w with the GMM methodology corresponds to 0.689 (≈ 0.7) with a standard error of 0.181, a t-value of 3.812, and a p-value of < 0.001 (test for $\hat{w} \neq 0$), hence the GMM methodology works. This holds for various as long as feasible parameter combinations and is also stable with respect to the starting value of w.

4.2 Campbell and Viceira (1999)

A more sophisticated approach is discussed by Campbell and Viceira (1999). The infinitely lived investor with Epstein-Zin-Weil utility wants to maximize his consumption and faces a time-varying equity premium and a constant riskless interest rate. His optimization problem is

$$u(c_t \mathbb{E}_t u_{t+1}) = \left((1-\delta) c_t^{(1-\rho)/\theta} + \delta(\mathbb{E}_t u_{t+1}^{1-\rho})^{1/\theta}\right)^{\theta/(1-\rho)} \tag{27}$$

where δ is the time preference or discount factor, $\theta = \frac{1-\rho}{1-\psi^{-1}}$, ψ^{-1} the elasticity of intertemporal substitution, and ρ the coefficient of relative risk aversion. The investor's one period portfolio return $r_{p,t+1}$ from time t to $t+1$ is defined as

$$r_{p,t+1} = w_t(r_{w,t+1} - r_f) + r_f \tag{28}$$

with w_t the allocation to the risky asset at time t with one period return $r_{w,t+1}$ when invested at time t and held through $t+1$, r_f the constant risk free rate, $r_{p,t+1}$ the portfolio return from t to $t+1$, $R_{t+1} = \log(r_{t+1})$, and $R_f = \log(r_f)$.

There exists one risk factor which follows an autoregressive process of order one

$$x_{t+1} = \mu + \phi(x_t - \mu) + \eta_{t+1} \tag{29}$$

where $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ is normally distributed with variance σ_η^2. The expected return of the risky asset is related to the risk factor

$$\mathbb{E}_t R_{t+1} = x_t + R_f \tag{30}$$

The unexpected return of the risky asset is

$$R_{t+1} = x_t + R_f + u_{t+1} \tag{31}$$

where $u_{t+1} \sim N(0, \sigma_u^2)$ is normally distributed with variance σ_u^2. The covariance between the shocks to the risk factor and the unexpected return is $\mathrm{cov}(u_{t+1}, \eta_{t+1}) = \sigma_{u\eta}$.

As shown by Campbell and Viceira (1999) the allocation to the risky asset w_t is dependent on two constants, a_0, a_1, and the risk factor x_t. Similarly the log-consumption-wealth ratio, $c_t - v_t$, is also dependent on the risk factor x_t and on three constants b_0, b_1, and b_2, where $C_t = \log(c_t), V_t = \log(v_t)$

$$w_t = a_0 + a_1 x_t \tag{32}$$

$$c_t - v_t = b_0 + b_1 x_t + b_2 x_t^2 \tag{33}$$

Assume that b_0, b_1, b_2, and the functional form of $c_t - v_t$ and w_t are known. Then a_0 and a_1 need to be estimated to determine

$$w_t = \bar{w}_t + \hat{a}_0 + \hat{a}_1 x_t \tag{34}$$

where $\bar{w}_t$ is some benchmark weight and $\hat{a}_0$ and $\hat{a}_1$ are the estimates of a_0 and a_1. This is exactly the formulation of the parametric approach for the asset class weights as presented above. Furthermore assume that $\psi^{-1} = \rho$ such that the Epstein-Zin-Weil utility reduces to the standard time-separable power utility with relative risk aversion. The sample analog for estimation is then

$$\max_{a_0,a_1} u(c) = \mathbb{E}\left[\sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t}\left(\frac{c_t^{1-\rho}}{1-\rho}\right)\right] = \max_{a_0,a_1} \frac{1}{N}\sum_{n=1}^{N}\left[\sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t}\left(\frac{c_t^{1-\rho}}{1-\rho}\right)\right] \tag{35}$$

The parameters used can be found in a working paper by Campbell and Viceira (1996) which is a more detailed version of Campbell and Viceira (1999) and correspond to the annual model 1890-1993 with $\rho = 4 = \psi^{-1}, \delta = 0.94, \phi = 0.798, \mu = 4.165E-2, \sigma_\eta^2 = 0.096E-2, \sigma_u^2 = 3.319E-2, \sigma_{u\eta} = -0.388E-2, R_f = 1.992E-2, b_0 = -2.995, b_1 = 1.249, b_2 = 9.62$ for a total of 500 draws with 100 observations each. Another 100 observations are discarded and used only for starting values due to the autoregressive formulation.

There are a couple of observations worth discussing.

- $\bar{w}_t = 0$. If the benchmark weight is ignored $\hat{a}_0 = 0.282 \approx a_0 = 0.301$ and $\hat{a}_1 = 10.7 \approx a_1 = 9.7$. The estimated coefficients are highly significant with p-values of 0 as expected. The variation can be explained with the random components in u and η as compared to the analytic solution by Campbell and Viceira.

- $\bar{w}_t = w_t^{CV}$. If the benchmark weight is the optimal solution as provided $\hat{a}_0 \neq a_0$ and $\hat{a}_1 \neq a_1$. $\hat{a}_0$ is insignificant at the 1% level and $\hat{a}_1$ is insignificant at the 5% level as expected.

- $\bar{w}_t = w_t^{CVopt}$. If the benchmark weight is the newly calculated optimal solution $\hat{a}_0 \neq a_0$ and $\hat{a}_1 \neq a_1$, and both are insignificant as expected. For calculating a_0 and a_1 in the formulation of Campbell and Viceira (1999) $\sigma_\eta^2, \sigma_u^2$ and $\sigma_{u\eta}$ are required. These parameters are prone to subtle but significant variation due to generating random u and η. The model a_0 and a_1 are therefore not absolutely robust to some variation - at least when b_0, b_1, and b_2 are not updated accordingly.

- $\bar{w}_t = c$. If the benchmark weight is any arbitrary constant, $\hat{a}_0$ and $\hat{a}_1$ are both significant as expected.

- $\bar{w}_t = \text{mean}(w_t^{CV})$ or $\bar{w}_t = \text{mean}(w_t^{CVopt})$. If the benchmark weight is equal to the mean of the optimal weights as provided or newly calculated $\hat{a}_0$ and $\hat{a}_1$ are both significant as expected.

Moreover, the results are stable with respect to input parameters to the optimization procedure. Therefore I conclude that the parametric approach is appropriate to be applied to the asset allocation problem.

5 Data

A fundamental problem to analyze asset allocation based on macroeconomic data is the lack of long time data series. This problem does not only relate to macroeconomic data but also to return series where large sample data is usually only available for the price index but not the total return index. Therefore I restrict the analysis to US markets which I take as proxy for the global allocation to stocks and bonds. The studies of Oertmann (1997) and Oertmann and Seiler (2010) serve as basis on the choice of the risk factors. Data is available from June 1964 and is split up into an in sample period, which lasts from June 1964 to December 2008, and into an out of sample period from January 2009 to December 2011. This corresponds to 570 end-of-month observations. The forecast period is one month. t refers to the last trading day of the month and the return period $t + 1$ refers to an investment into an asset class at t held until the end of next month, $t + 1$.

The risk free rate $r_{f,t+1}$ is the monthly equivalent to the annualized three month US government constant maturity rate as provided by Bloomberg. It would be more appropriate to take the one month US government constant maturity rate, however, this data series is only available starting in July 2001. The difference between the two annualized series is negligible with a mean in the difference of 0.06 %, a maximum difference of 0.33 % in November 2007, a standard deviation of the difference of 0.01 % and a correlation of 0.998. The risk free rate is the percentage return from t to $t + 1$ and is known at t.

The total return series are taken from CRSP data base. For stocks ($r_{s,t+1}$) this is the total return series of the value weighted CRSP stock market indexes (NYSE, AMEX, NASDAQ and ARCA). For bonds ($r_{b,t+1}$) I take the total return series of the CRSP US government bonds series with maturities 60 to 120 months. The total return series are the percentage rates from t to $t + 1$, hence monthly returns, but these are only known at $t + 1$.

Linking the financial asset allocation decision to macroeconomic variables is a challenging task as there is a great variety of potential variables to choose from. However, there are some criteria which limit the selection: (1) data should be available for a long time series, (2) data frequency should be at least monthly, (3) there should be a limited risk to significant post-release data revisions as for example with GDP, (4) the variables should have been discussed in the literature, and (5) the link between the variables and financial asset class returns should be intuitive.

These requirements are fulfilled for the term spread, credit spread and ted spread. Additionally I add common value parameters for stocks. All variables are discussed in Oertmann (1997) and Oertmann and Seiler (2010). In the following paragraphs I outline these variables and give a brief motivation for considering them in the model and an intuitive link to financial asset returns.

The term spread (term) is the difference between the 10 year and 3 month US government bond yield. The end-of-month-data are monthly averages of daily Bloomberg rates. A high and positive term spread or equivalently a steep interest curve is usually associated with an economy before the peak of its business cycle. This is the time when business activity grows strongest. As the business cycle matures the central bank raises target rates to limit the risk of inflation. In this way the term spread can be interpreted as indicator for the economic activity. Investors allocate their funds towards risky assets when they expect the economic activity to rise.

Another argument to invest into risky assets and specifically stocks is when trust into companies is high. The credit spread (credit) serves as a measure of trust and is the difference between corporate bonds with a rating of BAA and AAA (Moody's rating). The data is taken from CRSP and are monthly averages of the daily rates.

Risky assets get unattractive when the financial system is regarded unstable. This is measured by the ted spread (ted) which is the difference between the monthly averages of the three month interbank rate and the three month US government rate. Usually the USD Libor rate is taken as interbank rate. A drawback to this definition is that the Libor rate only starts in 1986. An alternative is the Eurodollar rate, which is available from 1971. To take advantage of a longer time series I rely on the three month certificate of deposit secondary market rate. The ted spread with the certificate of deposit rate is on average 0.11 basis points lower than the usual ted spread definition with the Libor rate. The standard deviation of this difference is 0.09. The reason for the lower rate is that certificate of deposits are insured by the Federal Deposit Insurance Corporation (FDIC) for banks and by the National Credit Union Administration (NCUA) for credit unions (Wikipedia, 2012). However, the correlation between the Libor ted spread and certificate of deposit ted spread is very high: 0.974 for the whole time series and even higher during the financial crisis 2008. This is reason enough to consider the ted spread definition with the certificate of deposit rate. The rate is taken from the Federal Reserve Economic Data (FRED) provided by the FED St. Louis.

Another argument to invest into risky assets and especially stocks is when they are cheap relative to their value. Two prominent value measures are the price earnings ratio (PE) and the dividend yield (DY). The dividend yield is the 12 month rolling difference known at t between the CRSP Stock Market Indexes including all distributions and the CRSP Stock Market Indexes excluding dividends. The price earnings ratio is the Shiller price earnings ratio downloaded from Mr. Shiller's website[1]. This price earnings ratio is measured for the S&P 500 and is taken as proxy for the CRSP stock universe. To ensure that the ratio is known at the end of the current month the price earnings ratio of the previous month is taken.

As discussed in section 2 the weight of the asset class could potentially depend on the asset class' past returns. Therefore I add a measure of past returns to the estimation defined as the difference between the return of the previous and the pre-previous period:

[1]http://www.econ.yale.edu/~shiller/data/ie_data.xls

$(r_{i,(t+1)-1} - r_{i,(t+1)-2}) \times 100$. Differently to the risk factors explained above the past returns are only considered for the relevant asset class, i.e. the past return of the asset class stocks (M_s) is only relevant for stocks, not for bonds, and vice versa in the case of bonds (M_b). This variable can also be interpreted as measuring a variant of momentum. However, the definition and application of momentum is different in this study compared to the literature. For example, in Asness, Moskowitz and Pedersen (2013) momentum is defined as the past 12-month cumulative raw return on the asset class skipping the last month's return. I will discuss both variants of momentum further down in section 6. Furthermore a constant for stocks c_s and bonds c_b added to the model.

Table 5 in Appendix A shows statistics of the data in and out of sample which shows significant differences between most variables.

6 Results

To discuss the performance of the model I compare it with a standard 60/40 portfolio strategy, i.e. a portfolio where 60% of the funds are held in stocks and 40% are held in bonds. These are also the benchmark weights $\bar{w}_{i,t}$ for stocks and bonds in equation 3.

Table 1 shows the mean results of various models with different combinations of intercept and past return added to the model or not, the dividend yield or Shiller PE or both and risk aversion parameters 1, 2, 5, 10, 15, and 20.

It is notable that the strategy outperforms the benchmark in terms of most parameters not only in sample but also out of sample. The Sharpe Ratio is almost constant across different values of risk aversion and is in sample 0.20 vs. 0.09 for the benchmark. Out of sample (0.60) the Sharpe Ratio is still almost double the Sharpe Ratio of the benchmark (0.38) and as well virtually constant across different levels of risk aversion. As the Sharpe Ratio measures the return relatively to the risk taken it is not surprising that the Ratio is independent of the risk aversion. The lower the risk aversion ρ the more risk is taken (σ_r) but also the expected return ($\bar{r}$) increases.

Although Jensen's Alpha is not fully appropriate to be used in an asset allocation context I report it in this study as it is widely regarded by practitioners. Jensen's Alpha measured relatively to the benchmark is significantly different from zero for the in sample period and is negatively correlated to the risk aversion. The same is true for the out of sample period where Jensen's Alpha is even larger. I calculate two versions of Jensen's Alpha out of sample: one where β is calculated in sample (β_i) and one where β is calculated out of sample (β_o). Jensen's Alpha is always significant out of sample unless a model specification with the dividend yield and without past returns is considered. In some rare cases Jensen's Alpha is insignificant when risk aversion is high. There is a notable

	In sample June 1964 - December 2008								
	SR		$\bar{r}$		σ_r		α %	Cert. Equ.	
ρ	s	bm	s	bm	s	bm	β_i	s	bm
1	0.20	0.09	7.3	0.7	31.0	2.9	^{s}6.05	4.00	0.70
2	0.21	0.09	3.8	0.7	15.2	2.9	^{s}2.85	2.08	0.65
5	0.21	0.09	1.8	0.7	6.2	2.9	^{s}1.14	1.08	0.52
10	0.21	0.09	1.1	0.7	3.1	2.9	^{s}0.57	0.76	0.29
15	0.19	0.09	0.9	0.7	2.1	2.9	^{s}0.36	0.63	0.05
20	0.19	0.09	0.8	0.7	1.6	2.9	^{s}0.27	0.57	-0.22

	Out of sample January 2009 - December 2011									
	SR		$\bar{r}$		σ_r		α %		Cert. Equ.	
ρ	s	bm	s	bm	s	bm	β_i	β_o	s	bm
1	0.55	0.38	12.2	1.2	22.6	3.1	^{s}8.81	^{s}8.80	9.90	1.14
2	0.65	0.38	8.5	1.2	13.2	3.1	^{s}6.48	^{s}5.14	6.88	1.09
5	0.61	0.38	3.1	1.2	5.0	3.1	^{s}2.23	^{s}1.77	2.46	0.95
10	0.62	0.38	1.6	1.2	2.6	3.1	^{s}1.27	^{s}0.92	1.29	0.70
15	0.58	0.38	1.0	1.2	1.7	3.1	^{s}0.68	^{s}0.52	0.76	0.46
20	0.58	0.38	0.8	1.2	1.4	3.1	^{s}0.53	^{s}0.39	0.57	0.21

Table 1: Average results of various parameter and risk aversion combinations. SR = Sharpe Ratio, $\bar{r}$ = mean return, σ_r = the standard deviation of the return, Cert. Equ. = Certainty Equivalent, s = strategy, bm = benchmark, ρ = risk aversion parameter, α % = Jensen's Alpha in % relative to the benchmark with β calculated in sample (β_i) or out of sample (β_o). ^{s} denotes a Jensen's Alpha significantly different from 0 with a p-value of ≤ 0.05. Values are monthly.

difference between Jensen's Alpha with β_i and β_o where the first is always larger compared to Jensen's Alpha with β_o. Note, that β_o is just known *after* the last realization of the out of sample period. This shows that there is a difference between the historically *expected* and *realized* Jensen's Alpha.

The certainty equivalent expresses a risk free rate of return such that an investor values this return equivalently to the expected utility of the return r_p of a risky portfolio: $u(C) = \mathbb{E}u(r_p)$, where C is the certainty equivalent. As expected the certainty equivalent is higher when ρ is small as the investor is ready to take more risk for a higher return. Moreover, the certainty equivalent of the strategy presented is always larger than the certainty equivalent of the benchmark - in sample and out of sample. Thus an investor requires a higher risk free rate in exchange to the strategy compared to the benchmark. With $\rho = 20$ the certainty equivalent is even negative for the benchmark in sample, hence a risk averse investor would tend not to follow the 60/40 strategy.

Table 2 shows the results of five selected models for a risk aversion level of 5 and 10. (1) The first model includes all variables (F), (2) the second model is similar to the full model apart from the constant excluded (A). (3) The next is a value model (V) with DY and PE as coefficients. The credit spread (credit) is dropped from the estimation as the coefficient is insignificant with DY and PE. (4) The fourth model considers past returns (C) and (5) the last model (M) is the same as C without the credit spread. In general the coefficients lose their significance when asset class specific coefficients (c_s, c_b) are allowed as can be seen when comparing models F with models A, V, C, and M.

As discussed above a measure for past return was added due to the formulation of the problem. However, the past return variable or momentum for stocks (M_s) is only significant if the dividend yield (DY) is used as value parameter. If an intercept (c_s and c_b) is taken into account M_s and M_b are always insignificant with c_b significant (model F). If c_s and c_b are considered without past returns only c_s is significant with DY as value variable and $\rho > 10$. c_s and c_b add overall little value to the model regardless of the specification. Therefore I will not consider c_s and c_b more detailed. I will first discuss the coefficients of stocks before analyzing the coefficients of bonds.

Regarding stocks the term spread (term) is always insignificant and the credit spread (credit) is only significant in model C when DY is included into the model and when $\rho > 10$ which is not shown in the table. The sign of the coefficient of credit is always negative when significant. This is intuitive: the lower the spread the higher the trust into companies and the more attractive are stocks.

The ted spread (ted) is significant regardless of the model specification and consistently negative, hence a high ted spread is associated with a low allocation to stocks. When the financial system is regarded unstable risk aversion picks up and risky investments as stocks get unattractive.

Surprisingly the value factors dividend yield (DY) and the Shiller price earnings ratio (PE) depend on ρ: PE tends to be significant when $\rho > 5$ and the DY tends to be significant when $\rho < 5$ (models A and V). The sign of the coefficient is as expected: DY is positive, hence the higher the dividend yield the higher the expected return of stocks. PE is negative, i.e. the lower the price earnings ratio the better for stocks, hence go long stocks when they are cheap.

Also the significance of M_s depends partly on ρ which tends to be generally significant when $\rho < 5$ or $\rho > 10$ unless an intercept is taken into account. When credit is included into the model (C) M_s is always significant. Note that values for $\rho < 5$ and $\rho > 10$ are not reported in the table. These results are weaker compared to the literature discussing value and momentum as for example Asness, Moskowitz and Pedersen (2013). Results do not change by a large factor in terms of performance when applying a standard momentum definition, i.e. the past 12-month cumulative raw return on the asset class skipping the last month's return. Interestingly, the momentum coefficients are then insignificant but the momentum variable on stock gets significant when considering transaction costs. However, the model is still clearly outperformed by the value model (V). Note that especially momentum is usually discussed in a strategy context, i.e. going long highly positive momentum assets and short highly negative momentum assets *within* an asset class. In this respect momentum is used differently compared to this study. Moreover, the purpose of considering momentum - or better past returns - in this paper is more of a technical nature.

Overall the allocation to bonds seems to be better predictable with respect to significance of the parameters. The term spread (term) is significant and consistently positive apart from the full model (F). The sign of the coefficient leaves room for interpretation: (1) The bond universe are long term US treasury bonds of maturities between 60 and 120 months. Long term bonds get usually more attractive when long term interest rates are high relative to short term interest rates and are not expected to rise further. (2) As short term interest rates are low this effect eventually feeds back to the long end, bringing down yields and increasing the total return for long term bond investors.

Different to the term spread the credit spread (credit) is always insignificant. The third macroeconomic variable ted spread (ted) is significant and positive for models V, C and M. As a measure of risk in the financial system this is intuitive: investors flee into US Treasuries.

Of the two value variables DY and PE only DY is of statistical relevance for bonds. The sign is negative and can be interpreted as relative attractiveness to invest into stocks. Unlike the past returns of stocks M_s, the past return variable for bonds M_b is always statistically significant unless an intercept is added to the model.

Table 3 shows detailed results of the models F, A, V, C and M for the in sample and out of sample periods. In sample there is not much difference between the models in terms of Sharpe Ratio. The mean return $\bar{r}$ is higher with models F, A, C, and M relative to V in sample, however also the risk borne (σ_r). Jensen's Alpha is always significantly positive. Models C and M outperform models F, A, and V with respect to Jensen's Alpha and the certainty equivalent.

Out of sample all strategies perform well compared to the benchmark strategy with M performing best compared to all models with respect to Sharpe Ratio and certainty equivalent. Jensen's Alpha is not significantly different from zero in case of model C with β_o and when ρ is large, however significant in all other cases. Model F clearly outperforms the other models with respect to Jensen's Alpha.

Because of the results discussed and the fact that credit is insignificant in the model with $\rho = 5$ and $\rho = 10$ model M dominates C with respect to significance of coefficients and parsimony. I will discuss a subtle drawback to strategy M over V further down stemming from M_s and M_b.

		stock							bond						
	ρ	term	credit	ted	DY	PE	M_s	c_s	term	credit	ted	DY	PE	M_b	c_b
F	5	0.11	0.04	-0.59 *	0.17	-0.01	0.12	0.07	1.00	-0.05	1.23	-0.77	0.02	0.63	-0.18 **
F	10	0.03	0.04	-0.34 **	0.05	-0.01 *	0.06	0.03	0.52 *	-0.08	0.63	-0.42	0.00	0.41	-0.09 **
A	5	0.13	-0.04	-0.54 *	0.16	-0.01	0.07		1.12 **	-0.49	1.64	-0.71 *	0.05	0.36 **	
A	10	0.04	-0.01	-0.32 **	0.05	-0.01	0.03		0.53 **	-0.22	0.76	-0.38 **	0.02	0.18 **	
V	5	0.10		-0.66 ***	0.15	-0.01			1.04 ***		2.39 ***	-0.89 ***	0.03		
V	10	0.03		-0.37 ***	0.04	-0.01 *			0.51 ***		1.16 ***	-0.47 ***	0.01		
C	5	0.18	-0.59	-0.52 **	0.24 *		0.08 **		1.41 ***	-0.67	2.65 **	-0.80 **		0.32 **	
C	10	0.08	-0.40	-0.29 **	0.09		0.04 **		0.68 ***	-0.22	1.27 **	-0.45 **		0.16 **	
M	5	0.03		-0.65 ***	0.14		0.06 *		1.24 ***		2.09 **	-0.85 ***		0.33 **	
M	10	-0.02		-0.39 ***	0.02		0.03		0.62 ***		1.04 **	-0.46 ***		0.17 **	

Table 2: Selection of test results. Full model (F), full model without constant (A), value (V), credit & momentum (C) and momentum (M). ρ = risk aversion parameter. *, ** and *** relate to a p-value of $\leq 0.10, \leq 0.05$ and ≤ 0.01. No star relates to insignificant coefficients.

As there are not yet leverage constraints in place the only limitation to leverage stems implicitly from ρ. The higher ρ the smaller the magnitude of the coefficients and the lower the weights to the risky asset classes as can be seen in table 2 and table 6 in Appendix A respectively. Thus the weights can get very large when ρ is small: With $\rho = 5$ the allocation to bonds reaches a maximum of 12 times the equity of the fund (100%). The average allocation to bonds is more reasonable with 70% to 110% of equity depending on which model is considered (table 6, Appendix A). Interestingly the out of sample allocation to stocks and bonds is well above the in sample allocation. Thus an investor following the strategy would have invested aggressively during this time which was ex post the right decision.

Figure 1 in Appendix A shows the asset allocation to stocks (black) and bonds (blue) for the in sample and out of sample period of the value strategy V for risk aversions $\rho = 5$ and $\rho = 10$. The beginning of the out of sample period is highlighted by a vertical red line. The horizontal green lines define 0% (no allocation) and 100% (full allocation of equity). If the allocation of all asset classes is in sum above 100%, i.e. above the upper horizontal green line the investor borrows money, if it is below the lower green line he is in total short risky assets and invests these funds into the riskless asset. Obviously the allocation into bonds has a higher volatility. Figure 1a shows the allocation when $\rho = 5$

		In sample June 1964 - December 2008								
		SR		$\bar{r}$		σ_r		α %	Cert. Equ.	
	ρ	s	bm	s	bm	s	bm	β_i	s	bm
F	5	0.24	0.09	2.0	0.7	6.3	2.9	^{s}1.36	1.07	0.52
	10	0.24	0.09	1.2	0.7	3.2	2.9	^{s}0.69	0.76	0.29
A	5	0.24	0.09	2.1	0.7	6.5	2.9	^{s}1.42	1.24	0.52
	10	0.24	0.09	1.3	0.7	3.3	2.9	^{s}0.70	0.83	0.29
V	5	0.20	0.09	1.8	0.7	6.5	2.9	^{s}1.14	1.07	0.52
	10	0.20	0.09	1.1	0.7	3.3	2.9	^{s}0.57	0.76	0.29
C	5	0.23	0.09	2.2	0.7	7.6	2.9	^{s}1.55	1.24	0.52
	10	0.23	0.09	1.3	0.7	3.9	2.9	^{s}0.77	0.83	0.29
M	5	0.23	0.09	2.1	0.7	7.1	2.9	^{s}1.43	1.19	0.52
	10	0.22	0.09	1.3	0.7	3.7	2.9	^{s}0.71	0.80	0.29

		Out of sample January 2009 - December 2011									
		SR		$\bar{r}$		σ_r		α %		Cert. Equ.	
	ρ	s	bm	s	bm	s	bm	β_i	β_o	s	bm
F	5	0.66	0.38	4.3	1.2	6.4	3.1	^{s}3.50	^{s}2.78	2.46	0.95
	10	0.67	0.38	2.3	1.2	3.4	3.1	^{s}1.88	^{s}1.41	1.29	0.70
A	5	0.56	0.38	2.8	1.2	4.9	3.1	^{s}2.75	^{s}2.34	2.24	0.95
	10	0.50	0.38	1.4	1.2	2.8	3.1	^{s}1.44	^{s}1.14	1.04	0.70
V	5	0.61	0.38	3.1	1.2	5.0	3.1	^{s}1.95	^{s}1.40	2.46	0.95
	10	0.62	0.38	1.6	1.2	2.6	3.1	^{s}1.06	^{s}0.70	1.29	0.70
C	5	0.56	0.38	2.8	1.2	4.9	3.1	^{s}2.31	^{s}1.93	2.24	0.95
	10	0.50	0.38	1.4	1.2	2.8	3.1	^{s}1.17	0.93	1.04	0.70
M	5	0.70	0.38	3.2	1.2	4.6	3.1	^{s}2.35	^{s}1.98	2.75	0.95
	10	0.69	0.38	1.8	1.2	2.5	3.1	^{s}1.27	^{s}1.01	1.47	0.70

Table 3: Detailed results of the models full (F), full model without constant (A), value (V), credit & momentum (C) and momentum (M). SR = Sharpe Ratio, $\bar{r}$ = mean return, σ_r = the standard deviation of the return, Cert. Equ. = Certainty Equivalent, s = strategy, bm = benchmark, ρ = risk aversion parameter, α % = Jensen's Alpha in % relative to the benchmark with β calculated in sample (β_i) or out of sample (β_o). ^{s} denotes a Jensen's Alpha significantly different from 0 with a p-value ≤ 0.05. Values are monthly.

and figure 1b shows the allocation when $\rho = 10$, respectively.

Figure 2 in Appendix A shows the asset allocation for strategy M again with $\rho = 5$ (figure 2a) and $\rho = 10$ (figure 2b). The largest drawback compared to strategy V is the high volatility of the weights around a mean allocation not much different to the weights in model V (figure 1). This is due to the past return variables M_s and M_b which are defined as $(r_{i,(t+1)-1} - r_{i,(t+1)-2}) \times 100$ and are by definition volatile: The mean of M_s and M_b is in sample roughly 0, the standard deviation is, however, 6.06 and 2.44 respectively (table 5, Appendix A). This ratio is large, much higher than for any other series.

Both, extreme weights and the tendency of the model to reallocate the portfolio often and by a large factor can be handled when including transaction or leverage costs without a large loss in performance. I will discuss this extension to the basic model in section 7.

Noteworthy there is a link between an economic crisis associated with a stock market crisis and the allocation to stocks: During the difficult economy and stock market period around and after the oil crisis in the 70s, the savings and loan crisis 1981, the Latin American debt crisis 1982, the excess at the end of the 1990s, the dot-com bubble and the financial crisis (partly out of sample) the allocation to stocks was low compared to other times.

7 Transaction and Leverage costs

One of the main drawbacks to the approach are the extreme weights and the tendency of the model to reallocate the portfolio often and by a large factor which makes an application challenging. No investor with risk aversion 5 applying model V or model M would reallocate 67% or even 168% respectively, of his portfolio on average every month. Neither would an average investor have the opportunity to lever his portfolio by a factor of 10 or even higher. However, this can be handled when including transaction costs or leverage costs.

Methodologically transaction and leverage costs are straight forward to include into the model. In case of asset class specific transaction costs c_i the absolute difference in weights need to be considered each period. Moreover, due to the change of security prices also the portfolio share might increase or decrease. If the allocation into asset i is held, the allocation to asset i at time t when allocating $w_{i,t-1}$ at $t-1$ is $w_{i,t}^h = w_{i,t-1}(1 + r_{i,t})$, where $w_{i,t}^h$ denotes the $(t-1)$ to t buy and hold allocation at t for asset i. Hence, the portfolio return $r_{p,t+1}$ in equation 8 changes to

$$\max_{\theta_i} \frac{1}{T} \sum_{t=0}^{T-1} [u(r_{p,t+1})] = \sum_{t=0}^{T-1} \left[u \left(\sum_{i=1}^{N} w_{i,t} r_{i,t+1} + w_{r_f,t} r_{f,t+1} - c_i |w_{i,t} - w_{i,t}^h| \right) \right] \tag{36}$$

Similar to transaction costs leverage costs s can be considered. This premium needs to be paid on top of the risk free rate. Therefore the lending and borrowing rate is different. To model transaction costs I use an indicator function: $I_s = 1$ if $\sum_{i=1}^{N} w_{i,t} > 1$ and $I_s = 0$ else.

$$\max_{\theta_i} \frac{1}{T} \sum_{t=0}^{T-1} [u(r_{p,t+1})] = \sum_{t=0}^{T-1} \left[u \left(\sum_{i=1}^{N} w_{i,t} r_{i,t+1} + w_{r_f,t} r_{f,t+1} - I_s s \left(\sum_{i=1}^{N} w_{i,t} - 1 \right) \right) \right] \quad (37)$$

Both approaches generate kinks in the first order conditions and the covariance matrix. However, as the asymptotic covariance matrix is calculated numerically the adaption of transaction and leverage costs is without risk for calculating significant values. Still, the kink in the first order condition can be challenging for the optimization procedure of the GMM algorithm. This is indeed the case for model M, where the optimization procedure fails when transaction costs get large. This is due to the large variation of the factors M_s and M_b. As expected the coefficients of M_s and M_b are close to 0 and insignificant when including transaction costs. The strategy outperforms the benchmark significantly in sample and out of sample and the average monthly reallocation reduces to 12% for stocks and to 64% for bonds when one way costs $c_i = 0.3\%$ and when risk aversion is 5. This compares to an average monthly reallocation of 52% for stocks and 116% for bonds when $c_i = 0\%$. However, when $c_i > 0.3\%$ the optimization qualitatively fails: M_s turns significant and both, M_s and M_b get large and the benchmark quickly outperforms the strategy.

Nonetheless, as there is qualitatively enough evidence that M_s and M_b turn insignificant when transaction costs are considered model V quickly outperforms model M. Recall that model V does not include M_s and M_b. Results for model V are very encouraging. The strategy outperforms the benchmark in sample and out of sample clearly and in terms of all performance measures. The Sharpe Ratio is even unchanged out of sample at around 0.56, regardless of the size of transaction costs. The allocation process is much smoother: The average reallocation in bonds shrinks from 55% when $c_i = 0\%$ to 27% when $c_i = 0.5\%$ and 11% when $c_i = 1.0\%$. The average reallocation in stocks remains constant around 11% which is not surprising, as the allocation to stocks was comparably smooth from the beginning. With higher transaction costs some coefficients of the asset class bonds get insignificant as the magnitude of coefficients get smaller. This does not necessarily mean that the connection between the risk factors and the asset allocation is lost but it is rather a result of the optimization. This conclusion also follows from the observation that the mean return stemming from the asset class bonds is not reduced by a large factor when including transaction costs. Only the standard deviation of the returns of bonds reduces substantially as transaction costs get larger.

The results clearly speak for themselves and are reported in detail in table 7 in appendix A for strategy V for different levels of c_i and risk aversion 5 and 10. Moreover, figure 8 in appendix A shows the effect to the asset allocation over time when including transaction costs.

When considering leverage costs the kink in the optimization is no problem: With higher leverage costs M_s and M_b get insignificant in model M quickly and the magnitude of coefficients reduces as well. This leads to an overall allocation to stocks and bonds of less or around 100% of the portfolio size. Leverage costs also cause the portfolio's average monthly reallocation to decrease. When $s = 5\%$ and risk aversion is 5 the average reallocation for stocks is 20% and for bonds 29%. Model M outperforms the benchmark in sample over all tested levels of transaction costs (up to 5%) and out of sample in terms of the Sharpe Ratio up to $s = 2\%$. Jensens's Alpha gets insignificant when $s \geq 1\%$ but is still positive and comparably large: When $s = 1\%$ Jensen's Alpha is 0.64 and when $s = 5\%$ Jensen's Alpha is 0.30.

Again, model V outperforms model M clearly when considering leverage costs. The Sharpe Ratio is in sample almost constant regardless of the size of s and is around 0.45 out of sample although leverage gets quickly unattractive. Jensen's Alpha is always significant even when $s = 5\%$. Similar to model M the average mean reallocation of stocks and bonds reduces substantially: from 12% when $s = 0\%$ to 8% when $s = 1\%$ and 6% when $s = 5\%$ for stocks and from 55% when $s = 0\%$ to 12% when $s = 1\%$ and 9% when $s = 5\%$ for bonds. The coefficients shrink in magnitude as s rises and turn insignificant. However the mean performance attribution from stocks and bonds remains constant, only the return variations are reduced in both asset classes. This leads to the same argument as above: The risk factors are still relevant to the asset allocation process, even if they "lose" significance. Detailed results are provided in table 8 in appendix A for strategy V for different levels of s and risk aversion 5 and 10. Figure 9 in appendix A shows the effect to the asset allocation when including leverage costs.

8 Robustness

It is convincing that the model performs well out of sample. Another question is whether the model is robust when altering the in sample period. This also answers the question whether a short in sample period would suffice, hence it would be better to adjust the model according to the current economy. Alternatively a large data sample is more important which would hint on persistent rules on how the economy is linked to the finance world.

For this I investigate models V and M discussed above. For model V, figure 3 in Appendix A shows the different coefficients (3a) and significance of these (3b) for different in sample periods. The x axis denotes the year on which the in sample period ends. The first observation in sample is June 1964 and the last month is November of the shown year, the first month of the out of sample period is December and the out of sample period lasts for 12 months.

Figure 3a Appendix A shows that the coefficients can be considered to be time stable with at least 20 years of in sample data (1984). This relates to $12 \times 20 = 240$ monthly observations or 30 observations for each of the 8 coefficients. There is a shift in the magnitude of coefficients around the financial crisis, which has, however, limited impact on returns and other measures of model fitness and does not challenge the stability of the method.

Moreover, it takes more than 30 years (1996) for the ted spread to get significant at a 5% level as can be seen from figure 3b. The other significant variables of model V are again significant with around 20 years of data. Risk factors which are not significant (table 2) are also not significant independent of the in sample period length. However, the p-values in figure 3b vary substantially for these variables, though whether the risk factor is insignificant with a p-value of 0.2 or 0.8 does not change the conclusion.

Similar to model V also the coefficients of model M require around 20 years of data to converge to stable values (figure 4a, Appendix A). Again, there is a slight shift of the coefficients around the financial crisis. For variables M_s (p-value $<$ 10%) and especially M_b (p-value $<$ 5%) a long time series is required to achieve the significance values as reported in table 2 (figure 4b, Appendix A). Results are only shown beginning in 1984. The first in sample month is again June 1964 and the last in sample month is December 2010.

Another test on the robustness is to consider a rolling in sample window and verify whether the coefficients and significance values are stable. For this I run a rolling regressions with a 30-year-in-sample-window. Results are shown in figure 5 in Appendix A. Although the coefficients are considerably stable, the shift in the magnitude of coefficients around the financial crisis of 2008 is more pronounced compared to the extending in sample period shown in figure 4. A larger change can be seen when looking at figure 5b in Appendix A which shows the significance of the coefficients. M_b is only significant at a 10% level in 2008 and 2009. M_s is significant at a 10% level during a number of years but not consistent. Moreover, the ted spread turns out to be insignificant for the years ending 2004 until 2007.

Considering the mean (μ_s) and standard deviation (σ_s) of measures of model of fitness of model M when extending the in sample period (figure 4), the Sharpe Ratio is very

		SR		$\bar{r}$		σ_r		α %		Cert. Equ.	
		s	bm	s	bm	s	bm	β_i	β_o	s	bm
20.5-29.5	μ	0.16	0.22	1.3	1.0	5.8	2.8	0.45	0.09	0.29	0.79
	σ	0.23	0.27	1.2	0.7	2.5	1.1	1.32	1.49	1.35	0.74
30.5-39.5	μ	0.28	0.34	1.5	0.9	4.5	2.6	0.71	0.42	0.98	0.68
	σ	0.33	0.57	1.5	1.0	1.3	0.9	1.03	0.84	1.58	1.03
40.5-46.5	μ	0.52	0.18	3.5	0.5	8.1	2.6	3.16	2.56	1.36	0.71
	σ	0.24	0.35	3.4	1.2	12.0	1.1	4.35	3.36	1.99	0.49

Table 4: Results of the one year out of sample performance of model momentum (M) with added in sample data and $\rho = 5$. Shown are the mean (μ) values of the performance measures for in sample periods 20.5-29.5 years (2), 30.5-39.5 years (3) and 40.5-46.5 years (4) and the associated σ. SR = Sharpe Ratio, $\bar{r}$ = mean return, σ_r = the standard deviation of the return, Cert. Equ. = Certainty Equivalent, s = strategy, bm = benchmark, ρ = risk aversion parameter, α % = Jensen's Alpha in % relative to the benchmark with β calculated in sample (β_i) or out of sample (β_o). Values are monthly.

stable ($\mu_s = 0.25, \sigma_s = 0.01$ vs. benchmark $\mu_b = 0.10, \sigma_b = 0.02$), the certainty equivalent ($\mu_s = 1.29, \sigma_s = 0.08$ vs. $\mu_b = 0.58, \sigma_b = 0.05$), and Jensen's Alpha ($\mu_s = 1.43, \sigma_s = 0.19$) are stable and converge as expected to the values as shown in table 3.

Table 4 shows a more detailed analysis for the out of sample periods where the first out of sample period begins January 1985 and ends December 1985 and the last out of sample period begins January 2011 and ends November 2011. There are three blocks: 20.5 to 29.5 years of in sample data, 30.5 to 39.5 years of in sample data, and 40.5 to 46.5 years of in sample data. The strategy does not outperform the benchmark out of sample with respect to the Sharpe Ratio and the certainty equivalent in the window 20.5-29.5, however Jensen's Alpha is still positive and the strategy also beats it's benchmark with respect to return. For the window 30.5-39.5 only the Sharpe Ratio is slightly worse than the benchmark, still the strategy outperforms with respect to return, Jensen's Alpha and the certainty equivalent. In the last window 40.5-46.5 the strategy outperforms the benchmark clearly.

This can also be seen from figure 6 in Appendix A which shows the linked logarithmic out of sample performance since 1985 of the strategy (black) and the benchmark (blue). Within the first years the strategy does not outperform the benchmark and volatility is comparably high. The performance of the strategy improves over time with a massive spike in 2008 which results from a high allocation to bonds during the financial crisis. Especially the last years show considerable outperformance relative to the benchmark. Such a spike also occurs in 1982 along with the 2008 spike in model V.

Noteworthy, the out of sample performance of model M of the 30 year rolling in sample strategy compared to the strategy which includes more and more in sample data is virtually similar (figure 7, Appendix A). Only after the financial crisis the 30 year rolling in sample window slightly outperforms the strategy with more and more in sample data added.

A more detailed analysis than changing the last in sample year is to change the last in sample month. In fact, there is some variation in coefficients but not significance around the second half of 2008. This is especially true for the ted spread coefficient for the asset class bonds which can also be seen when examining the yearly change in coefficients in figure 3a and figure 4a in Appendix A. This has some, however not sizeable impact on returns and a limited effect on the model's performance measure.

These observations are evidence enough to argue that a larger data sample adds value to the model where in any case at least 30 years of data should be considered to ensure stable coefficients.

9 Conclusion

Building up on Brandt, Santa-Clara and Valkanov (2009) I propose a novel approach to solve an investor's asset allocation problem. The investor maximizes his time separable utility with choosing the right mix of asset classes. Different to other asset allocation approaches not the returns, but the weights of the assets are driven by risk factors. I focus on US stocks and long term US Treasury bonds as asset classes and the term spread, credit spread and ted spread as global macro risk factors. Moreover, I consider the dividend yield and Shiller's Price Earnings ratio as global value factors as well as asset class specific past returns which can be interpreted as momentum. The risk free rate serves as residual to the allocation to bonds and stocks such that the weights sum always up to 100%.

To verify the method I test the generalized method of moment estimator and the parametric asset allocation approach on two well known models: Merton (1969) and Campbell and Viceira (1999). As the moment conditions can not be estimated in a closed form I use a numerical method for estimation. This induces the risk of getting stuck in local maxima, i.e. different initial solutions result in different final solutions. To solve this problem I suggest an iterative optimization procedure.

The results are very encouraging and the method works fine in sample and out of sample. A standard 60/40 stock-bond-portfolio serves as benchmark. The benchmark is greatly outperformed in terms of Sharpe Ratio and certainty equivalent. Jensen's Alpha is always significantly positive in sample. Only in some model specifications Jensen's Alpha tends to get insignificant out of sample especially when ρ is large but is significantly positive else.

Both, the coefficients and the performance measures are directly related to the risk aversion coefficient ρ: The higher ρ the lower the magnitude of the coefficients, hence the magnitude of the weights and the lower the mean of the return, standard deviation, Jensen's Alpha and certainty equivalent. The outperformance over the benchmark is challenged but still present for levels of ρ as high as 20. The Sharpe Ratio does not vary with ρ which is reasonable as it is a measure of how much risk was taken for the return achieved.

Three models are discussed in more detail: a value model with the term spread, credit spread, dividend yield and price earnings ratio; a moment model with the term spread, credit spread, ted spread, dividend yield and the asset class specific momentum variable and another moment model including the credit spread. The signs of the coefficients are meaningful and intuitive.

Testing for robustness I verify that the coefficients converge to the values calculated. For this I add yearly in sample data such that the in sample period gets larger each year. The coefficients are stable with at least 20 years of data. With at least 30 years of data the same holds true for the significance apart from the momentum variables which require a longer time series to get significant. The financial crisis induces a shock to the estimation of the coefficients in the second half of 2008 which is, however, of minor relevance to the performance measures and the return series. I also compare the added-in-sample-strategy to a 30-year-rolling in sample data window and find that the out of sample performance is almost similar. In general there is enough evidence that a large data sample adds value to the model, hence enduring rules link the economy to finance.

The major drawback to the strategy, excessive leverage and large allocation changes each period, can conveniently be handled when including transaction or leverage costs. In both cases the mean reallocation is quickly reduced without losing a lot of performance. The value model clearly dominates all other models. In the case of leverage costs the portfolio exposure quickly shrinks below or around 100%. This makes the strategy applicable.

Possible extensions could be to add other asset classes as commodities, hedge funds or private equity and non US stock and bonds to the model. The feasibility depends mainly on the availability of economic and financial data which is limited for most regions, especially emerging markets and asset classes as hedge funds and private equity. Furthermore the inclusion of other asset classes requires a well defined total return series. Another interesting variation of the model is to focus on strategies as value or carry rather than asset classes.

This model can add valuable objectivity to the tactical asset allocation process in portfolio management. In this sense this model can not only contribute to a deeper understanding of the links between the economy and finance but also improve asset management.

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A Appendix

In sample June 1964 - December 2008

	$r_f \times 100$	r_s	r_b	term	credit	ted	DY	PE	M_s	M_b
Min	0.003	-0.225	-0.074	-2.65	0.32	0.04	0.84	6.62	-20.66	-14.24
25%	0.336	-0.018	-0.003	0.33	0.74	0.27	2.33	11.63	-3.86	-1.35
Med	0.429	0.011	0.005	1.42	0.91	0.45	3.20	18.50	-0.27	0.01
μ	0.464	0.008	0.006	1.37	1.03	0.68	3.36	19.50	0.00	0.01
75%	0.585	0.038	0.015	2.49	1.23	0.86	4.31	24.59	3.70	1.23
Max	1.210	0.166	0.109	4.42	3.38	4.70	7.36	44.66	27.53	10.24
σ	0.216	0.045	0.018	1.28	0.45	0.63	1.36	8.69	6.06	2.44

Out of sample January 2009 - December 2011

	$r_f \times 100$	r_s	r_b	term	credit	ted	DY	PE	M_s	M_b
Min	-0.001	-0.100	-0.035	1.97	0.76	0.09	1.41	13.06	-12.02	-7.00
25%	0.004	-0.020	-0.004	2.60	0.99	0.15	2.16	18.07	-4.91	-1.57
Med	0.010	0.020	0.005	3.14	1.13	0.18	2.41	20.04	-2.74	0.21
μ	0.009	0.016	0.006	3.01	1.37	0.28	2.43	19.60	-0.08	-0.10
75%	0.013	0.060	0.019	3.40	1.34	0.31	2.61	21.84	2.86	1.42
Max	0.021	0.114	0.034	3.70	3.09	0.91	3.54	23.71	19.89	5.66
σ	0.005	0.055	0.016	0.50	0.66	0.22	0.52	2.98	7.95	2.60

Table 5: Statistical summary data

		in sample						out of sample					
		stock			bond			stock			bond		
	ρ	min	μ	max	min	μ	max	min	μ	max	min	μ	max
F	5	-2,2	0,7	2,6	-4,5	1,0	9,5	-1,1	0,9	2,3	-1,2	2,8	5,3
	10	-1,2	0,3	1,3	-2,3	0,5	4,8	-0,4	0,5	1,2	-0,6	1,5	2,7
A	5	-2,0	0,6	2,4	-4,8	1,1	8,7	-0,4	0,9	2,0	0,3	2,7	4,9
	10	-1,1	0,3	1,2	-2,4	0,5	4,3	-0,1	0,5	1,0	0,2	1,4	2,4
V	5	-2,2	0,6	1,7	-3,5	1,1	12,0	0,3	0,8	1,2	1,7	2,7	4,4
	10	-1,2	0,3	0,8	-1,8	0,5	5,9	0,2	0,5	0,6	0,9	1,3	2,2
C	5	-2,6	0,7	2,5	-4,9	0,8	11,5	-1,7	0,8	2,0	0,8	2,5	4,5
	10	-1,5	0,4	1,3	-2,5	0,4	5,9	-1,0	0,4	1,1	0,4	1,4	2,5
M	5	-2,4	0,7	2,1	-4,9	0,7	10,6	-0,3	0,9	2,0	0,7	2,7	5,3
	10	-1,3	0,4	1,0	-2,5	0,4	5,4	-0,1	0,5	1,0	0,4	1,4	2,8

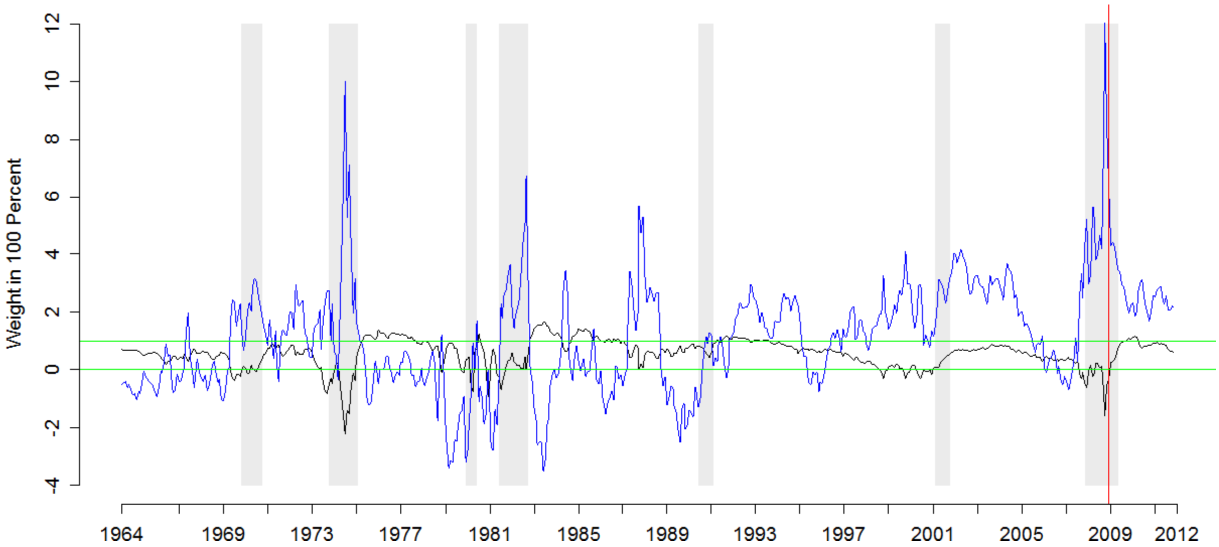
Table 6: Weights of selected test results in 100%. Minimum (min), mean (μ) and maximum (max) weights for the models F, A, V, C and M with risk aversion $\rho = 5$ and $\rho = 10$ for stocks and bonds in and out of sample.

		In sample June 1964 - December 2008												Out of sample January 2009 - December 2011											
		SR		$\bar{r}$		σ_r		α %	Cert. Equ.		turnover s		SR		$\bar{r}$		σ_r		α %		Cert. Equ.		turnover s	
	c_i	s	bm	s	bm	s	bm	β_i	s	bm	stock	bond	s	bm	s	bm	s	bm	β_i	β_o	s	bm	stock	bond
	0%	0.20	0.09	1.8	0.7	6.5	2.9	^{s}1.14	1.07	0.52	0.12	0.56	0.56	0.38	2.7	1.2	4.9	3.1	^{s}1.95	^{s}1.40	2.19	0.95	0.07	0.32
	0.1%	0.20	0.09	1.7	0.7	6.1	2.9	^{s}1.01	1.00	0.52	0.12	0.50	0.56	0.38	2.6	1.2	4.6	3.1	^{s}1.83	^{s}1.31	2.10	0.95	0.06	0.29
	0.2%	0.19	0.09	1.5	0.7	5.6	2.9	^{s}0.89	0.94	0.51	0.12	0.45	0.56	0.37	2.5	1.2	4.4	3.1	^{s}1.72	^{s}1.23	2.01	0.94	0.06	0.26
	0.3%	0.18	0.09	1.4	0.7	5.2	2.9	^{s}0.79	0.89	0.51	0.11	0.39	0.56	0.37	2.3	1.2	4.2	3.1	^{s}1.62	^{s}1.15	1.93	0.94	0.06	0.23
	0.4%	0.18	0.09	1.3	0.7	4.9	2.9	^{s}0.70	0.84	0.51	0.11	0.33	0.56	0.37	2.2	1.2	3.9	3.1	^{s}1.52	^{s}1.08	1.85	0.93	0.06	0.20
$\rho = 5$	0.5%	0.17	0.09	1.2	0.7	4.5	2.9	^{s}0.62	0.80	0.51	0.11	0.28	0.56	0.37	2.1	1.2	3.7	3.1	^{s}1.43	^{s}1.02	1.77	0.93	0.06	0.17
	0.6%	0.16	0.09	1.2	0.7	4.3	2.9	^{s}0.55	0.76	0.51	0.11	0.23	0.57	0.37	2.0	1.2	3.5	3.1	^{s}1.35	^{s}0.97	1.70	0.93	0.06	0.15
	0.7%	0.16	0.09	1.1	0.7	4.0	2.9	^{s}0.50	0.73	0.51	0.11	0.19	0.57	0.37	1.9	1.2	3.3	3.1	^{s}1.27	^{s}0.92	1.63	0.92	0.06	0.13
	0.8%	0.15	0.09	1.0	0.7	3.9	2.9	^{s}0.45	0.69	0.50	0.11	0.15	0.58	0.37	1.8	1.2	3.1	3.1	^{s}1.20	^{s}0.88	1.57	0.92	0.06	0.11
	0.9%	0.14	0.09	1.0	0.7	3.7	2.9	^{s}0.41	0.66	0.50	0.11	0.11	0.58	0.37	1.7	1.2	2.9	3.1	^{s}1.13	^{s}0.84	1.51	0.91	0.06	0.09
	0.10%	0.13	0.09	1.0	0.7	3.7	2.9	^{s}0.37	0.63	0.50	0.11	0.09	0.59	0.36	1.6	1.2	2.8	3.1	^{s}1.07	^{s}0.80	1.45	0.91	0.06	0.08
		SR		$\bar{r}$		σ_r		α %	Cert. Equ.		turnover s		SR		$\bar{r}$		σ_r		α %		Cert. Equ.		turnover s	
	c_i	s	bm	s	bm	s	bm	β_i	s	bm	stock	bond	s	bm	s	bm	s	bm	β_i	β_o	s	bm	stock	bond
	0%	0.20	0.09	1.1	0.7	3.6	2.9	^{s}0.57	0.76	0.29	0.07	0.28	0.57	0.38	1.5	1.2	2.6	3.1	^{s}1.06	^{s}0.70	1.16	0.70	0.03	0.16
	0.1%	0.20	0.09	1.1	0.7	3.0	2.9	^{s}0.51	0.73	0.29	0.06	0.25	0.57	0.38	1.4	1.2	2.5	3.1	^{s}1.00	^{s}0.65	1.12	0.70	0.03	0.14
	0.2%	0.19	0.09	1.0	0.7	2.8	2.9	^{s}0.45	0.70	0.29	0.06	0.22	0.56	0.37	1.3	1.2	2.3	3.1	^{s}0.95	^{s}0.61	1.07	0.70	0.03	0.13
	0.3%	0.18	0.09	0.9	0.7	2.6	2.9	^{s}0.40	0.67	0.28	0.06	0.19	0.56	0.37	1.3	1.2	2.2	3.1	^{s}0.90	^{s}0.57	1.03	0.69	0.03	0.11
	0.4%	0.18	0.09	0.9	0.7	2.4	2.9	^{s}0.35	0.65	0.28	0.06	0.16	0.57	0.37	1.2	1.2	2.1	3.1	^{s}0.85	^{s}0.54	0.99	0.69	0.03	0.10
$\rho = 10$	0.5%	0.17	0.09	0.9	0.7	2.3	2.9	^{s}0.31	0.63	0.28	0.06	0.14	0.57	0.37	1.2	1.2	2.0	3.1	^{s}0.80	^{s}0.51	0.96	0.69	0.03	0.09
	0.6%	0.16	0.09	0.8	0.7	2.2	2.9	^{s}0.28	0.61	0.28	0.06	0.11	0.57	0.37	1.1	1.2	1.9	3.1	^{s}0.76	^{s}0.48	0.92	0.68	0.03	0.07
	0.7%	0.15	0.09	0.8	0.7	2.1	2.9	^{s}0.25	0.59	0.28	0.06	0.09	0.58	0.37	1.1	1.2	1.8	3.1	^{s}0.73	^{s}0.46	0.89	0.68	0.03	0.06
	0.8%	0.15	0.09	0.8	0.7	2.0	2.9	^{s}0.23	0.57	0.28	0.06	0.07	0.58	0.37	1.0	1.2	1.7	3.1	^{s}0.96	^{s}0.44	0.86	0.67	0.03	0.05
	0.9%	0.14	0.09	0.7	0.7	1.9	2.9	^{s}0.20	0.55	0.27	0.06	0.06	0.58	0.37	1.0	1.2	1.6	3.1	^{s}0.66	^{s}0.42	0.84	0.67	0.03	0.05
	0.10%	0.13	0.09	0.7	0.7	1.9	2.9	^{s}0.18	0.53	0.27	0.06	0.04	0.58	0.36	0.9	1.2	1.6	3.1	^{s}0.63	^{s}0.40	0.81	0.67	0.03	0.04

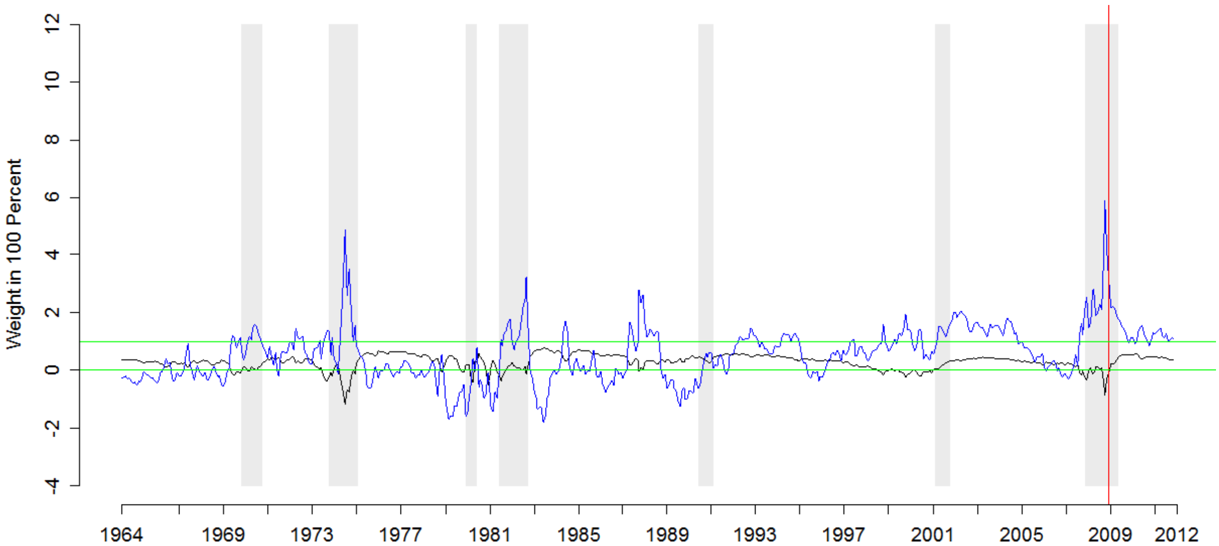
Table 7: Model V for various levels of one way transaction costs (c_i) and risk aversion $\rho = 5, 10$. SR = Sharpe Ratio, $\bar{r}$ = mean return, σ_r = the standard deviation of the return, Cert. Equ. = Certainty Equivalent, s = strategy, bm = benchmark, α % = Jensen's Alpha in % relative to the benchmark with β calculated in sample (β_i) or out of sample (β_o). ^{s} denotes a Jensen's Alpha significantly different from 0 with a p-value of ≤ 0.05, turnover s = average turnover of strategy. Values are monthly.

		In sample June 1964 - December 2008												Out of sample January 2009 - December 2011													
		SR		$\bar{r}$		σ_r		$\alpha\%$	Cert. Equ.		turnover s		SR		$\bar{r}$		σ_r		α %		Cert. Equ.		turnover s	
	$I_s s$	s	bm	s	bm	s	bm	β_i	s	bm	stock	bond	s	bm	s	bm	s	bm	β_i	β_o	s	bm	stock	bond
	0%	0.20	0.09	1.8	0.7	6.5	2.9	^{s}1.14	1.07	0.52	0.12	0.56	0.56	0.38	2.7	1.2	4.9	3.1	^{s}1.95	^{s}1.40	2.19	0.95	0.07	0.32
	0.1%	0.20	0.09	1.5	0.7	5.1	2.9	^{s}0.85	0.98	0.52	0.11	0.42	0.52	0.38	2.0	1.2	3.8	3.1	^{s}1.36	0.88	1.65	0.95	0.06	0.24
	0.2%	0.20	0.09	1.2	0.7	3.6	2.9	^{s}0.59	0.90	0.52	0.09	0.23	0.51	0.38	1.5	1.2	2.9	3.1	^{s}0.95	0.55	1.29	0.95	0.05	0.14
	0.3%	0.19	0.09	1.1	0.7	3.5	2.9	^{s}0.56	0.88	0.52	0.10	0.22	0.47	0.38	1.3	1.2	2.7	3.1	^{s}0.80	0.41	1.12	0.95	0.05	0.13
	0.5%	0.19	0.09	1.1	0.7	3.2	2.9	^{s}0.50	0.84	0.52	0.09	0.18	0.45	0.38	1.1	1.2	2.3	3.1	^{s}0.59	0.29	0.93	0.95	0.05	0.11
$\rho = 5$	1.0%	0.19	0.09	1.0	0.7	2.7	2.9	^{s}0.42	0.81	0.52	0.08	0.12	0.43	0.38	1.0	1.2	2.3	3.1	^{s}0.54	0.20	0.88	0.95	0.05	0.08
	1.5%	0.19	0.09	1.0	0.7	2.5	2.9	^{s}0.38	0.80	0.52	0.08	0.12	0.46	0.38	0.9	1.2	1.9	3.1	^{s}0.46	0.23	0.82	0.95	0.04	0.07
	2.0%	0.19	0.09	1.0	0.7	2.6	2.9	^{s}0.37	0.79	0.52	0.08	0.10	0.46	0.38	0.9	1.2	2.0	3.1	^{s}0.45	0.22	0.84	0.95	0.04	0.07
	2.5%	0.19	0.09	0.9	0.7	2.2	2.9	^{s}0.33	0.77	0.52	0.06	0.09	0.40	0.38	1.0	1.2	2.4	3.1	^{s}0.55	0.10	0.84	0.95	0.04	0.06
	3.0%	0.19	0.09	0.9	0.7	2.1	2.9	^{s}0.31	0.77	0.52	0.06	0.09	0.41	0.38	1.0	1.2	2.4	3.1	^{s}0.51	0.10	0.84	0.95	0.04	0.06
	5.0%	0.19	0.09	0.9	0.7	2.1	2.9	^{s}0.30	0.77	0.52	0.06	0.09	0.47	0.38	0.8	1.2	1.7	3.1	^{s}0.33	0.21	0.74	0.95	0.03	0.06
		SR		$\bar{r}$		σ_r		$\alpha\%$	Cert. Equ.		turnover s		SR		$\bar{r}$		σ_r		α %		Cert. Equ.		turnover s	
	$I_s s$	s	bm	s	bm	s	bm	β_i	s	bm	stock	bond	s	bm	s	bm	s	bm	β_i	β_o	s	bm	stock	bond
	0%	0.20	0.09	1.1	0.7	3.6	2.9	^{s}0.57	0.76	0.29	0.07	0.28	0.57	0.38	1.5	1.2	2.6	3.1	^{s}1.06	^{s}0.53	1.16	0.70	0.03	0.16
	0.1%	0.20	0.09	1.0	0.7	2.8	2.9	^{s}0.49	0.74	0.29	0.06	0.23	0.54	0.38	1.2	1.2	2.3	3.1	^{s}0.87	^{s}0.44	1.00	0.70	0.03	0.14
	0.2%	0.20	0.09	0.9	0.7	2.3	2.9	^{s}0.40	0.72	0.29	0.06	0.17	0.55	0.38	1.1	1.2	2.0	3.1	^{s}0.75	0.39	0.91	0.70	0.03	0.10
	0.3%	0.20	0.09	0.9	0.7	2.4	2.9	^{s}0.40	0.72	0.29	0.06	0.18	0.53	0.38	1.0	1.2	1.9	3.1	^{s}0.69	0.34	0.84	0.70	0.03	0.11
	0.5%	0.20	0.09	0.9	0.7	2.1	2.9	^{s}0.36	0.71	0.29	0.06	0.15	0.53	0.38	0.9	1.2	1.7	3.1	^{s}0.61	0.29	0.77	0.70	0.03	0.09
$\rho = 10$	1.0%	0.20	0.09	0.9	0.7	1.9	2.9	^{s}0.32	0.69	0.29	0.06	0.12	0.52	0.38	0.9	1.2	1.7	3.1	^{s}0.57	^{s}0.29	0.75	0.70	0.03	0.07
	1.5%	0.20	0.09	0.8	0.7	1.8	2.9	^{s}0.29	0.69	0.29	0.05	0.10	0.53	0.38	0.8	1.2	1.6	3.1	^{s}0.53	^{s}0.29	0.73	0.70	0.03	0.06
	2.0%	0.20	0.09	0.8	0.7	1.7	2.9	^{s}0.27	0.68	0.29	0.05	0.08	0.54	0.38	0.8	1.2	1.6	3.1	^{s}0.53	^{s}0.29	0.73	0.70	0.03	0.05
	2.5%	0.20	0.09	0.8	0.7	1.7	2.9	^{s}0.28	0.68	0.29	0.05	0.09	0.53	0.38	0.8	1.2	1.5	3.1	^{s}0.48	^{s}0.44	0.70	0.70	0.03	0.06
	3.0%	0.20	0.09	0.8	0.7	1.7	2.9	^{s}0.27	0.68	0.29	0.05	0.09	0.50	0.38	0.8	1.2	1.7	3.1	^{s}0.50	0.23	0.70	0.70	0.03	0.06
	5.0%	0.20	0.09	0.8	0.7	1.7	2.9	^{s}0.27	0.68	0.29	0.05	0.09	0.50	0.38	0.8	1.2	1.5	3.1	^{s}0.45	0.23	0.67	0.70	0.03	0.05

Table 8: Model V for various levels of one way leverage costs ($I_s s$) and risk aversion $\rho = 5, 10$. SR = Sharpe Ratio, $\bar{r}$ = mean return, σ_r = the standard deviation of the return, Cert. Equ. = Certainty Equivalent, s = strategy, bm = benchmark, α % = Jensen's Alpha in % relative to the benchmark with β calculated in sample (β_i) or out of sample (β_o). ^{s} denotes a Jensen's Alpha significantly different from 0 with a p-value of ≤ 0.05, turnover s = average turnover of strategy. Values are monthly.

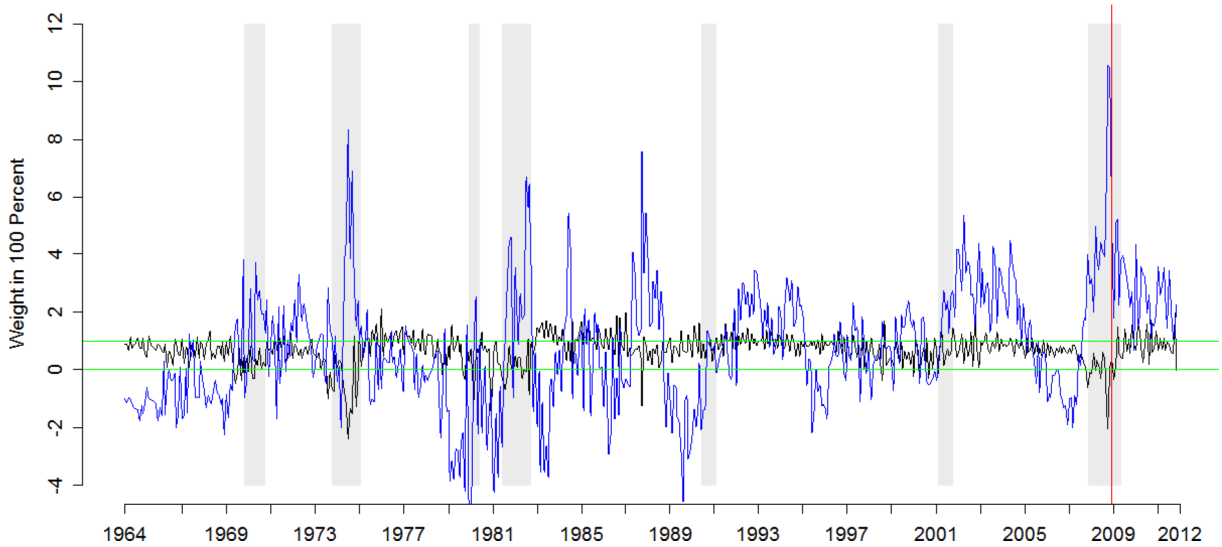


(a) $\rho = 5$

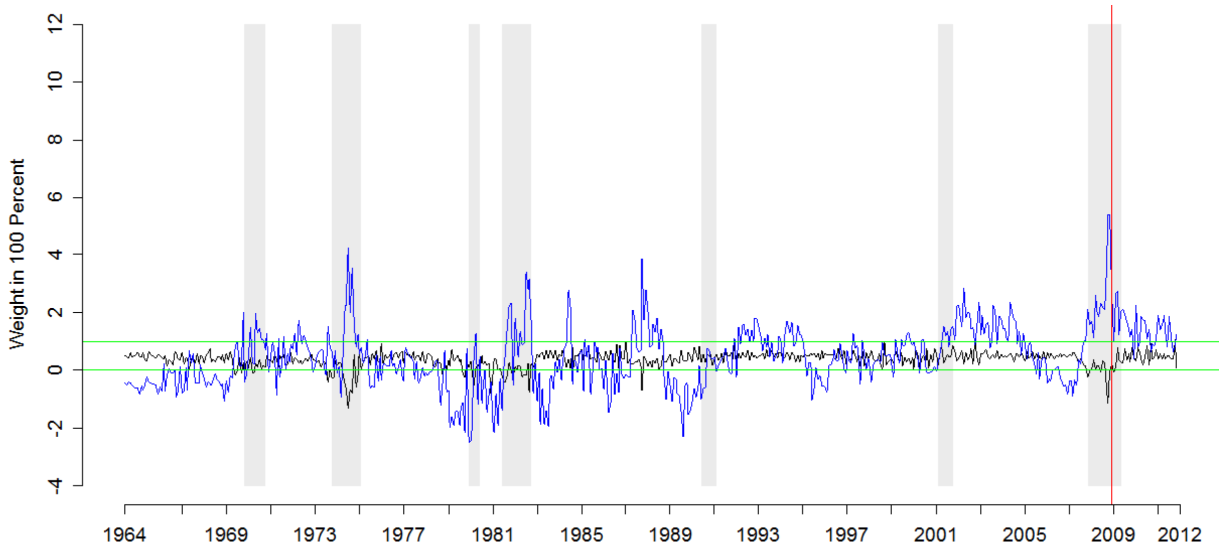


(b) $\rho = 10$

Figure 1: Asset allocation of model V. Black = allocation to stocks, blue = allocation to bonds. The red vertical line denotes the begin of the out of sample period. The lower horizontal green line symbolizes no allocation (0%), the upper green line full allocation of equity (100%) to the asset class. The gray shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St. Louis.

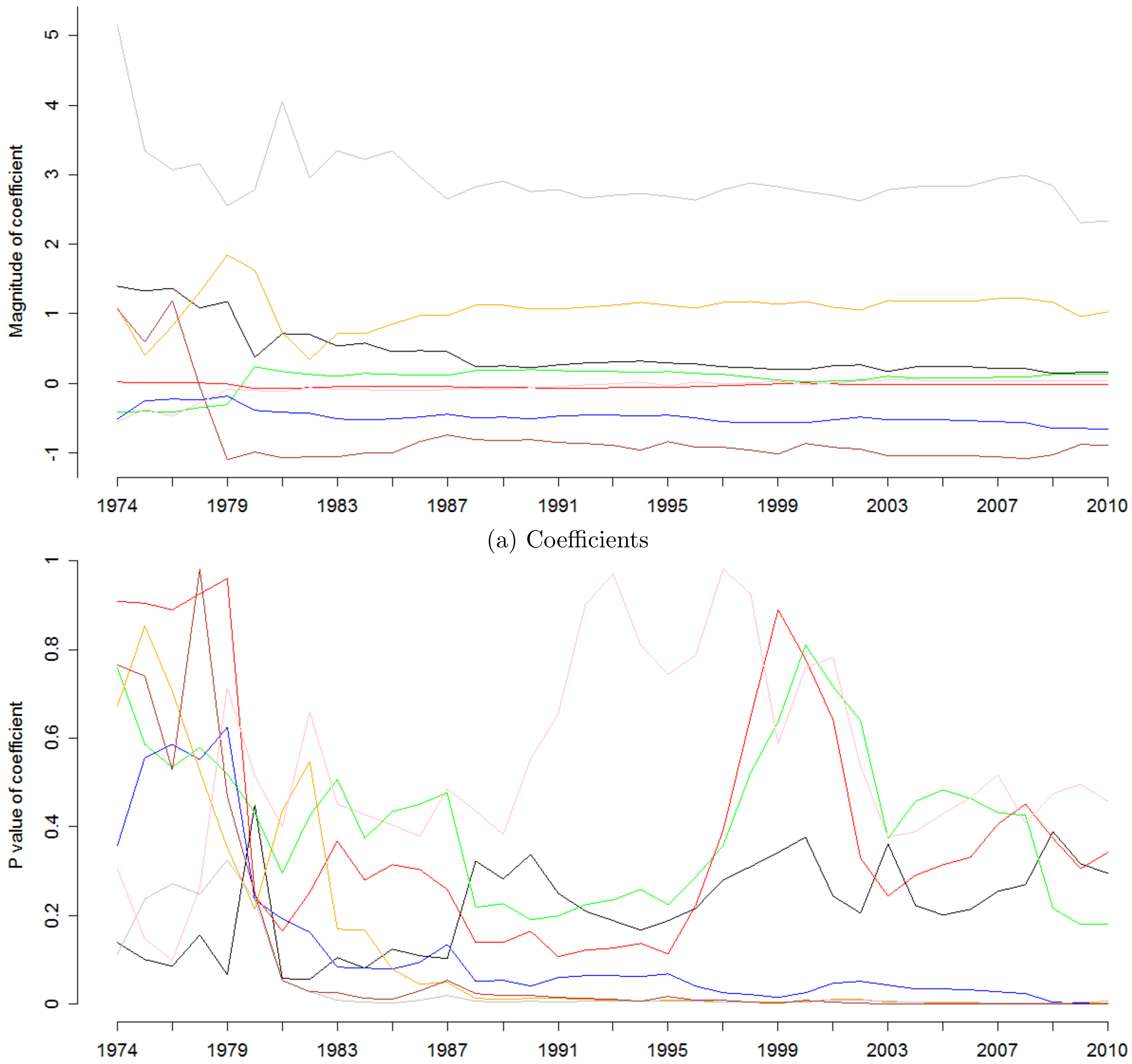


(a) $\rho = 5$



(b) $\rho = 10$

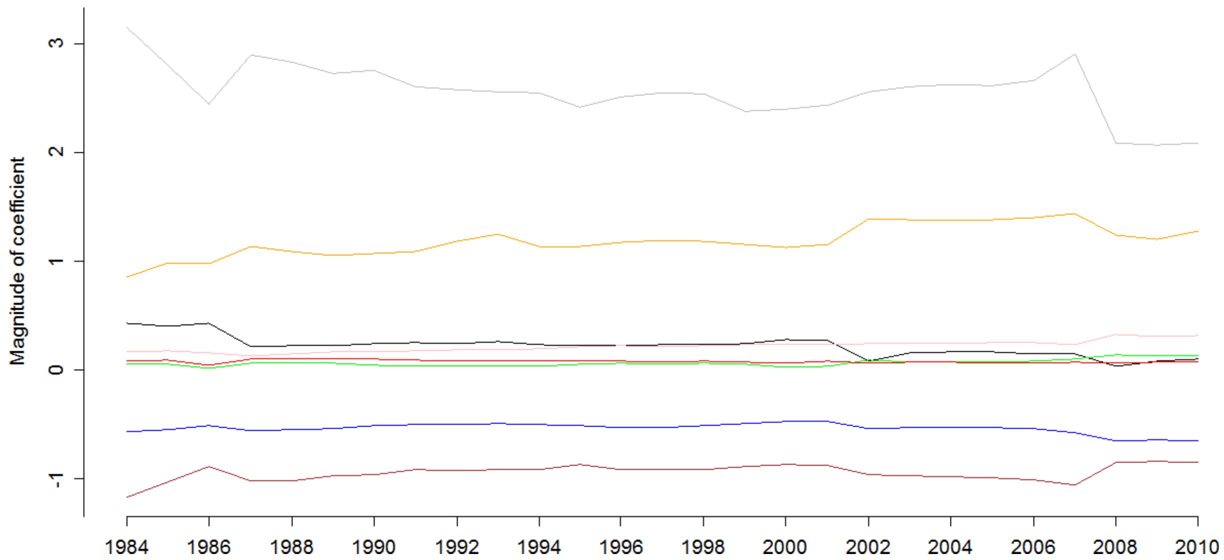
Figure 2: Asset allocation of model M. Black = allocation to stocks, blue = allocation to bonds. The red vertical line denotes the begin of the out of sample period. The lower horizontal green line symbolizes no allocation (0%), the upper green line full allocation of equity (100%) to the asset class. The gray shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St. Louis.



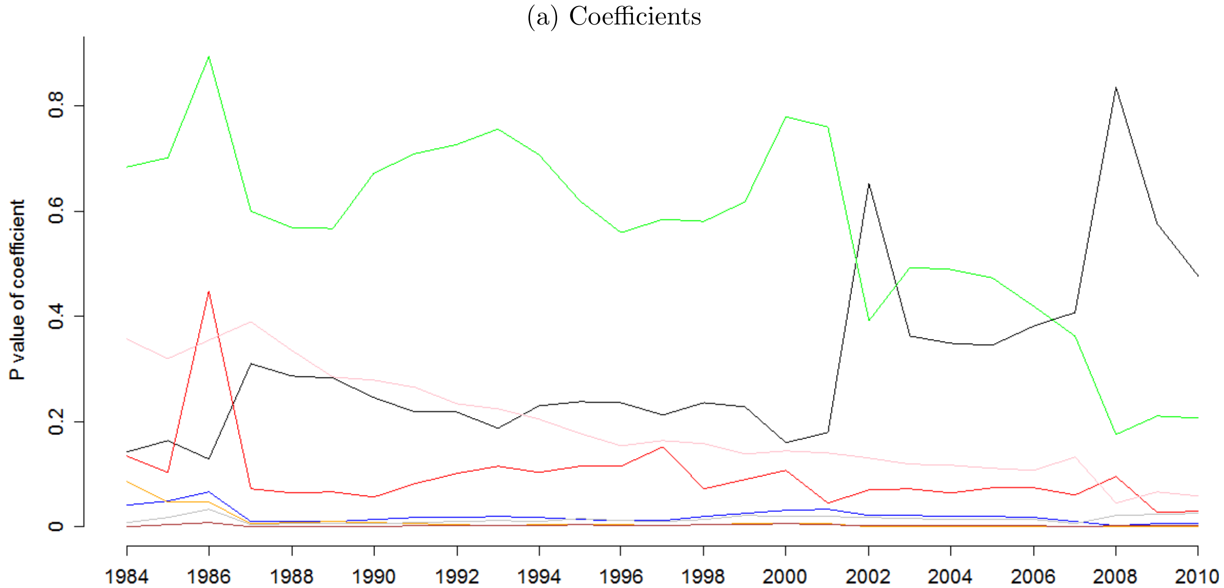
(a) Coefficients

(b) Significance

Figure 3: Robustness of coefficients. Model V. The x axis denotes the year on which the in sample period ends where the first month is June 1964 and the last month is November of the year shown. Stocks: black = term, blue = ted, green = DY, red = PE. Bonds: orange = term, gray = ted, brown = DY, pink = PE.

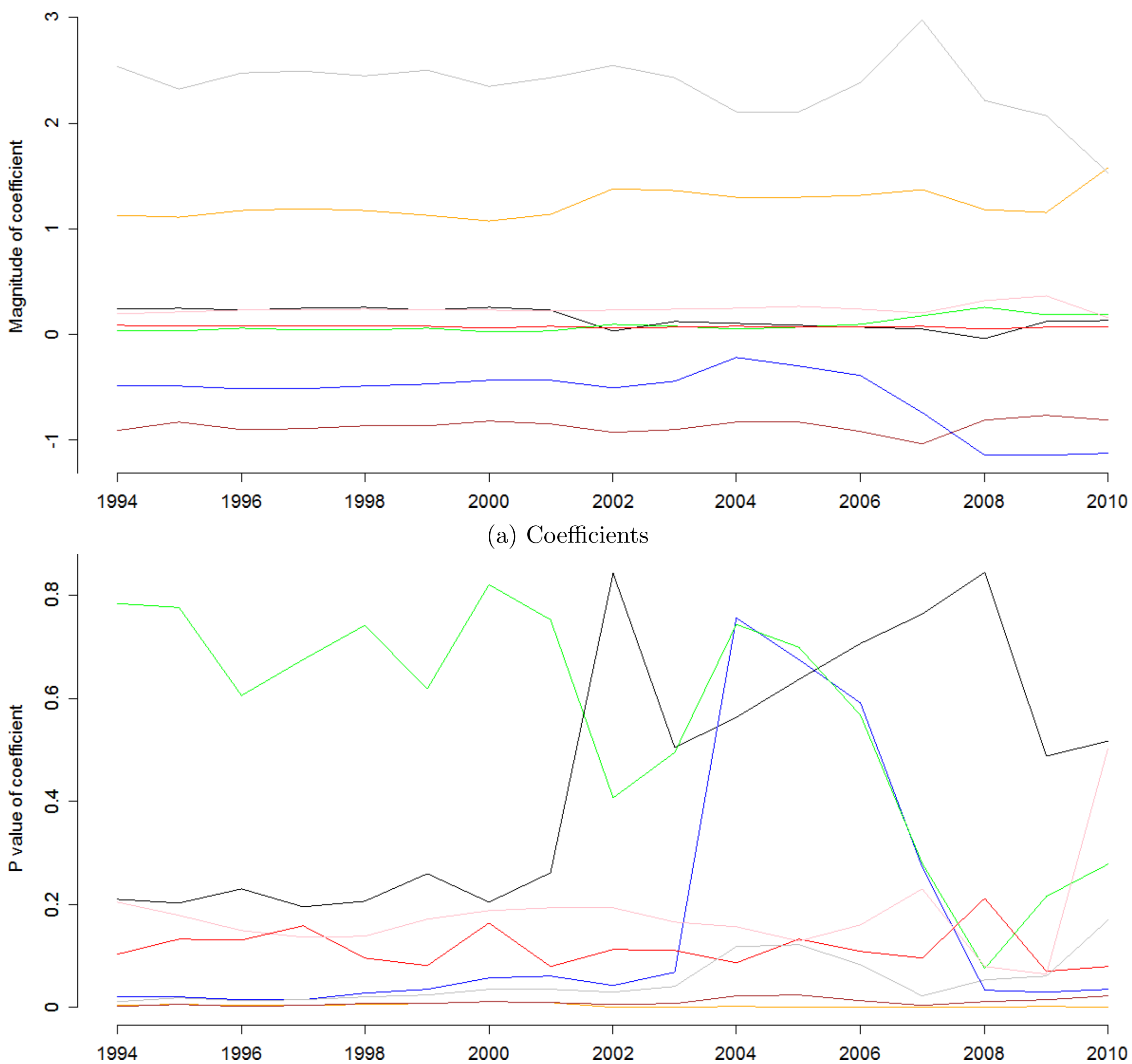


(a) Coefficients



(b) Significance

Figure 4: Robustness of coefficients. Model M. The x axis denotes the year on which the in sample period ends where the first month is June 1964 and the last month is December of the year shown. Stocks: black = term, blue = ted, green = DY, red = M_s. Bonds: orange = term, gray = ted, brown = DY, pink = M_b.



(a) Coefficients

(b) Significance

Figure 5: Robustness of coefficients, 30 year rolling window. Model M. The x axis denotes December of the year on which the in sample period ends. The first month is 30 years before. Stocks: black = term, blue = ted, green = DY, red = M_s. Bonds: orange = term, gray = ted, brown = DY, pink = M_b.

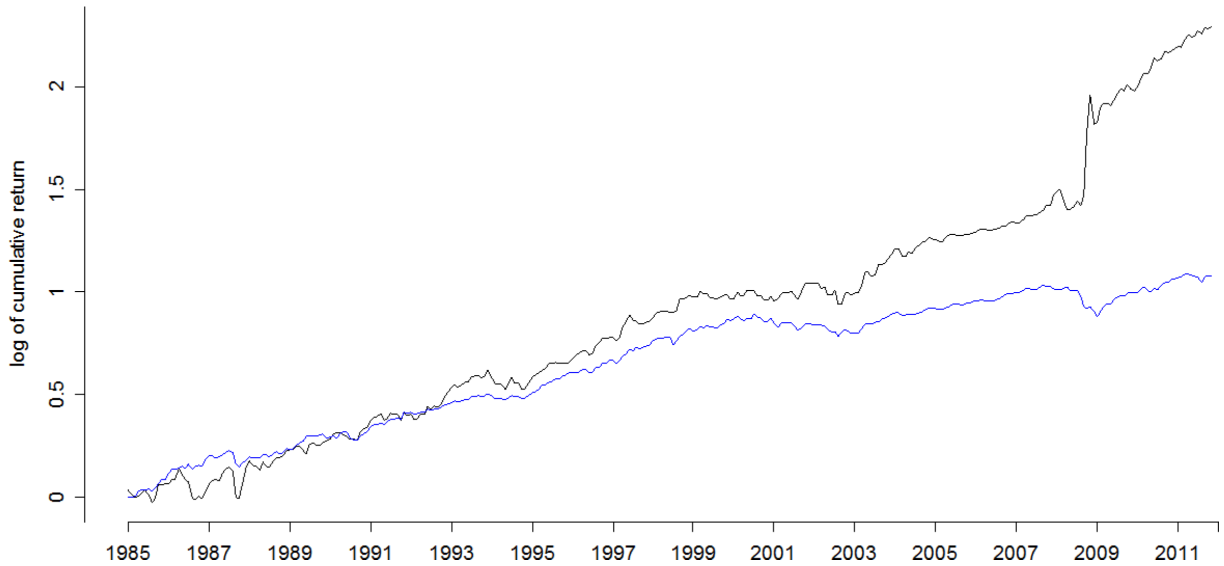


Figure 6: Linked one year out of sample performance since 1985 of model M. Black = strategy, blue = benchmark.

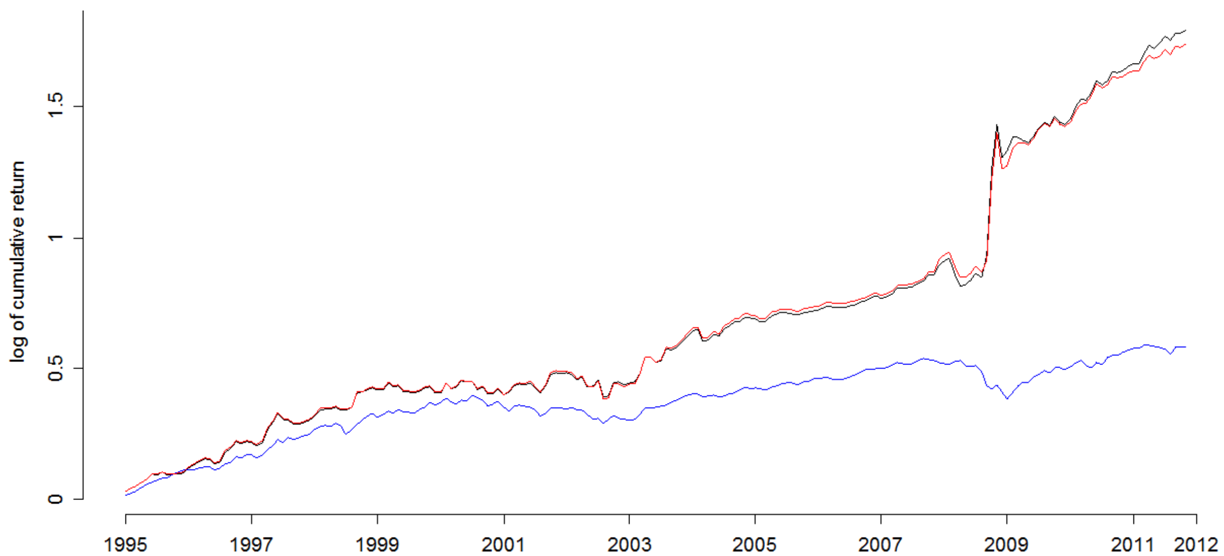
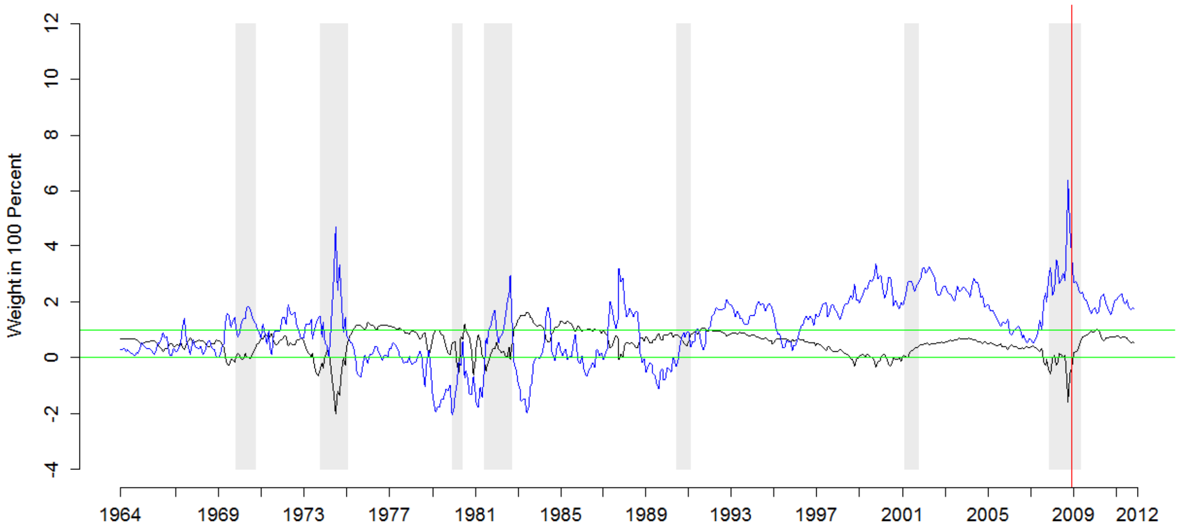
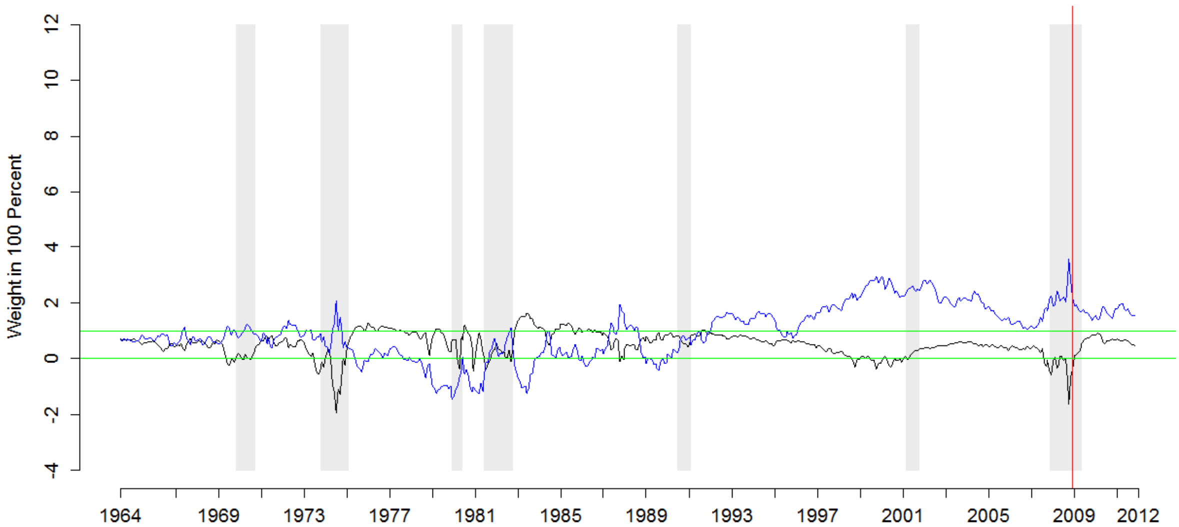


Figure 7: Comparison of the linked one year out of sample performance since January 1995 of strategy M. Black = 30 year rolling in sample window, red= in sample period extended each year, blue = benchmark.

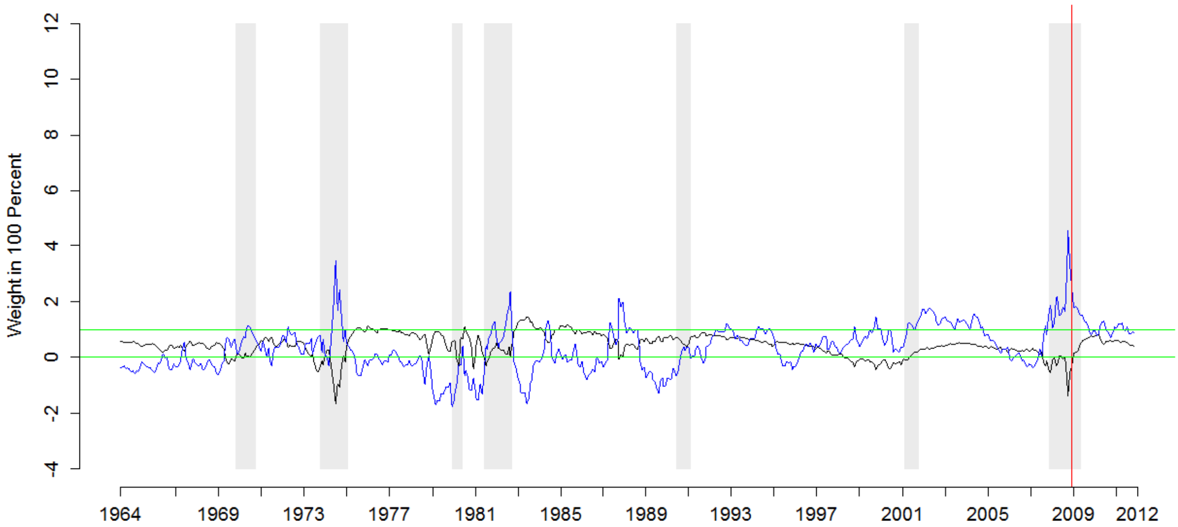


(a) 0.5% one way transaction costs, $\rho = 5$

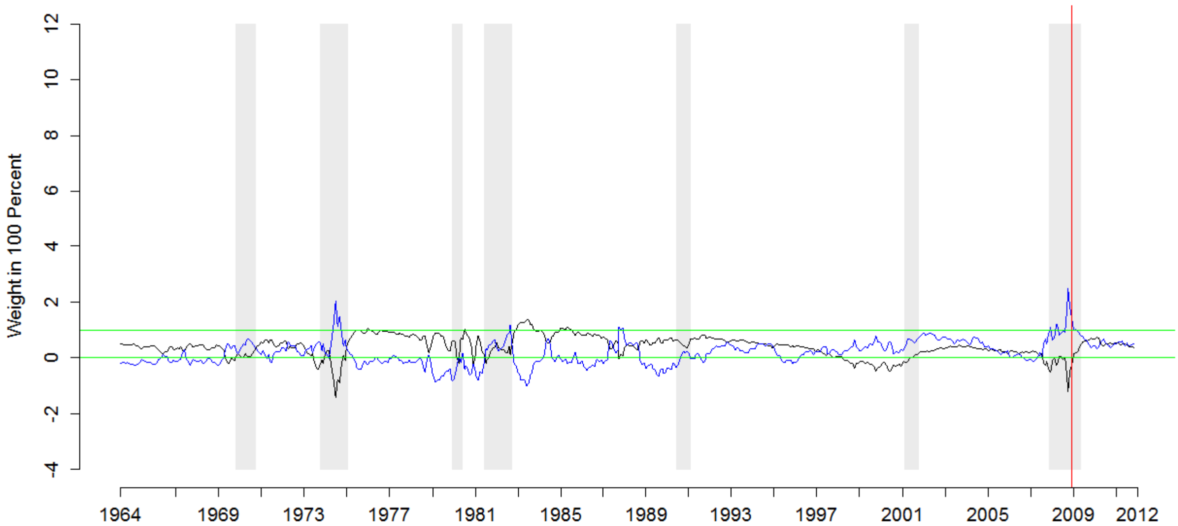


(b) 0.8% one way transaction costs, $\rho = 5$

Figure 8: Asset allocation of model V with risk aversion $\rho = 5$ and one way transaction costs of 0.5% and 0.8%. Black = allocation to stocks, blue = allocation to bonds. The red vertical line denotes the begin of the out of sample period. The lower horizontal green line symbolizes no allocation (0%), the upper green line full allocation of equity (100%) to the asset class. The gray shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St. Louis.



(a) 0.3% leverage costs, $\rho = 5$



(b) 1.0% leverage costs, $\rho = 5$

Figure 9: Asset allocation of the model V with risk aversion $\rho = 5$ and leverage costs of 0.3% and 1.0%. Black = allocation to stocks, blue = allocation to bonds. The red vertical line denotes the begin of the out of sample period. The lower horizontal green line symbolizes no allocation (0%), the upper green line full allocation of equity (100%) to the asset class. The gray shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St. Louis.